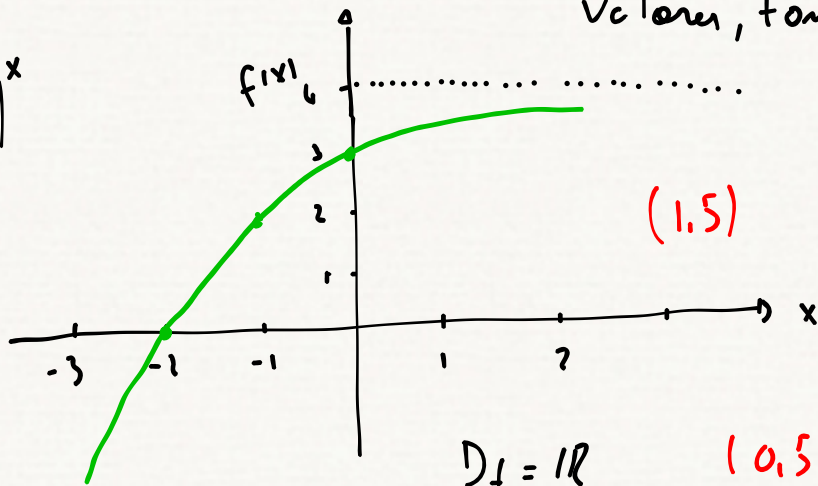


# GABARITO PE, 5/5/2023

Q1)  $f(x) = a - \left(\frac{1}{2}\right)^x = a - 2^{-x}$ , a assumir alguns valores, tome  $a=4$ .

$$f(x) = 4 - \left(\frac{1}{2}\right)^x$$



$$D_f = \mathbb{R} \quad (0,5)$$

$$I_f = (-\infty, 4) \quad (1,0)$$

Q2)

1) Correção que dobra o capital em 2 anos:

$$f(t=2 \text{ anos}) = 2$$

$$f(t) = \underline{\underline{2^{t/2}}}, \quad t \text{ em anos.} \quad (1,0)$$

2) Taxa equivalente anual:  $f(t=1) = (1+t_a)$

$$\Rightarrow 2^{1/2} = 1+t_a \Rightarrow t_a = \underline{\underline{1 - \sqrt{2}}} \quad (1,0)$$

3) Quando o capital triplica?

$$f(t) = 3 \Rightarrow 2^{t/2} = 3$$

$$\Rightarrow \log_2 2^{t/2} = \log_2 3 \Rightarrow t = \underline{\underline{2 \log_2 3}} \quad (1,0)$$

(pode-se usar outras bases para o log)

Q3  $z = -2\sqrt{2} + 2\sqrt{2}i$  (há uma variação com  $z = -3\sqrt{2} + 2\sqrt{2}i$ ).

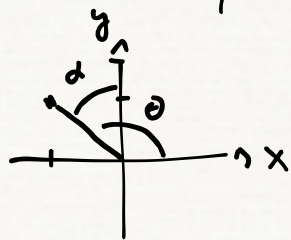
1)  $z - 2\bar{z} = ?$

$-2\bar{z} = 4\sqrt{2} + 4\sqrt{2}i, z - 2\bar{z} = -2\sqrt{2} + 2\sqrt{2}i + 4\sqrt{2} + 4\sqrt{2}i$

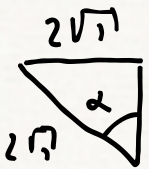
$\Rightarrow z - 2\bar{z} = 2\sqrt{2} + 6\sqrt{2}i$  (1,0)

2)  $|z| = \sqrt{z \cdot \bar{z}} = \sqrt{4 \cdot 2 + 4 \cdot 2} = \sqrt{16} = 4$  (1,0)

3) Forma polar:  $z = |z| (\cos \theta + i \operatorname{sen} \theta)$



$z = 4 \left( \cos\left(\frac{3\pi}{4}\right) + i \operatorname{sen}\left(\frac{3\pi}{4}\right) \right)$  (1,0)



$\theta = \frac{\pi}{2} + \alpha = \frac{3\pi}{4}$

$\hookrightarrow \operatorname{tg} \alpha = 1 \Rightarrow \alpha = \frac{\pi}{4}$

4)  $z^4$ ?  $z^n = |z|^n (\cos(n\theta) + i \operatorname{sen}(n\theta))$

$\Rightarrow z^4 = 4^4 \left( \cos\left(4 \cdot \frac{3\pi}{4}\right) + i \operatorname{sen}\left(4 \cdot \frac{3\pi}{4}\right) \right)$

$= 4^4 \left( \underbrace{\cos(3\pi)}_{-1} + i \underbrace{\operatorname{sen}(3\pi)}_0 \right)$

$z^4 = -4^4$  (1,0)