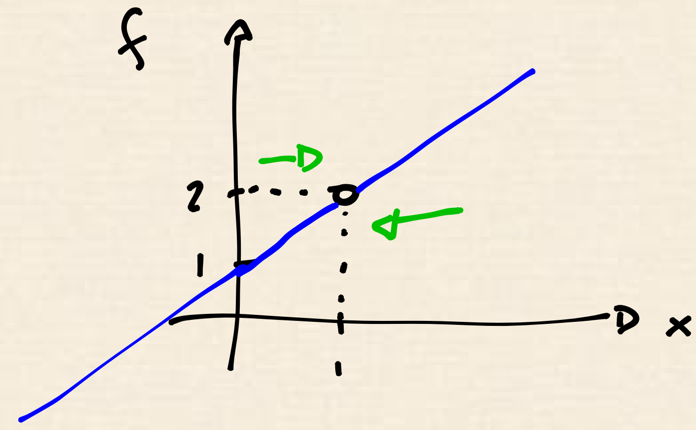
10/9/2020

Ex. 1 $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$, $x^2 - 1 = (x+1)(x-1)$



$$\frac{x^2-1}{x-1} = \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = x+1$$

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} x + 1 = 2$$

//

Ex. 2 $f(x) = \dfrac{x^2 + 3x - 4}{x^2 - 3x + 2}$, $\lim\limits_{x \to 1} f$? , $\lim\limits_{x \to 2} f$?

Raízes do denominador: $x^2 - 3x + 2 = 0$

$$x = \frac{1}{2}\left[3 \pm \sqrt{9 - 4 \cdot 2}\right] \begin{cases} \to x_1 = 2 \\ \to x_2 = 1 \end{cases}$$

$\underbrace{\lim\limits_{x \to 1}}\, f \sim \text{"}\dfrac{0}{0}\text{"}$?

$$x^2 - 3x + 2 = \underbrace{(x-1)}(x-2)$$

Divisão do numerador por $(x-1)$:

$$\begin{array}{l|l} x^2 + 3x - 4 & x - 1 \\ \hline -x^2 + x & x + 4 \\ \quad\; 4x - 4 & \\ \quad -4x + 4 & \\ \hline \qquad\; 0 & \end{array} \Rightarrow x^2 + 3x - 4 = (x-1)(x+4)$$

$$f(x) = \frac{\cancel{(x-1)}(x+4)}{\cancel{(x-1)}(x-2)} = \frac{x+4}{x-2}, \quad \lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x+4}{x-2} = \frac{5}{-1} = -5$$

//

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{x+4}{x-2} = \text{"}\frac{6}{0}\text{"} \quad \nexists.$$

Ex.3 $\lim_{x\to 0} \dfrac{\sqrt{x^2+100}-10}{x^2} = \text{"}\dfrac{0}{0}\text{"}$?

$$\frac{\sqrt{x^2+100}-10}{x^2}\cdot\frac{\sqrt{x^2+100}+10}{\sqrt{x^2+100}+10} = \frac{x^2+\cancel{100}+\cancel{10\sqrt{x^2+100}}-\cancel{10\sqrt{x^2+100}}-\cancel{100}}{x^2\left(\sqrt{x^2+100}+10\right)}$$

$$= \frac{\cancel{x^2}}{\cancel{x^2}\left(\sqrt{x^2+100}+10\right)} = \frac{1}{\sqrt{x^2+100}+10}$$

$$\lim_{x\to 0}\frac{\sqrt{x^2+100}-10}{x^2} = \lim_{x\to 0}\frac{1}{\sqrt{x^2+100}+10} = \frac{1}{20}\;//$$

Ex.4 $\lim_{x\to 0}\dfrac{\text{sen}(6x)}{3x}$?

$$\boxed{\lim_{x\to 0}\frac{\text{sen}(x)}{x} = 1}$$

$$= \lim_{x\to 0}\frac{2\cdot\text{sen}(6x)}{6x} = 2\lim_{u\to 0}\frac{\text{sen}(u)}{u}, \quad u = 6x,\ x\to 0,\ u\to 0$$

$$= 2\;//$$

Ex.5 $$\lim_{x\to 0}\frac{\text{sen}(2x)}{\text{sen}(5x)} = \lim_{x\to 0}\frac{\frac{\text{sen}(2x)}{2x}\cdot 2\cancel{x}}{\frac{\text{sen}(5x)}{5x}\cdot 5\cancel{x}} = \frac{2}{5}\lim_{x\to 0}\frac{\text{sen}(2x)/2x}{\text{sen}(5x)/5x}$$

$$= \frac{2}{5}\,\frac{\lim_{x\to 0}\text{sen}(2x)/2x}{\lim_{x\to 0}\text{sen}(5x)/5x} = \frac{2}{5}\cdot\frac{1}{1} = \frac{2}{5}\;//$$

Limites Laterais

Exercício Sec. 2, 1.

$$f(x) = \begin{cases} -x, & x < 0 \\ 1, & 0 < x < 1 \\ x^2, & x \geq 1 \end{cases}$$

$$\lim_{x \to 0^-} f(x), \lim_{x \to 0^+} f(x), \lim_{x \to 1^-} f(x), \lim_{x \to 1^+} f(x) \quad ?$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (-x) = 0$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (1) = 1$$

$$\lim_{x \to 0^-} f \neq \lim_{x \to 0^+} f \Rightarrow \lim_{x \to 0} f \not\exists$$

- - - - - - - - - - - - - - - - - - - -

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (1) = 1$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 = 1$$

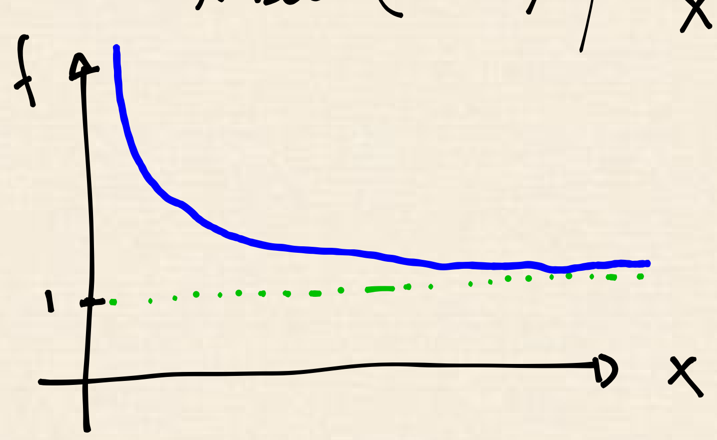
$$\lim_{x \to 1^-} f = \lim_{x \to 1^+} f = 1 \Rightarrow \lim_{x \to 1} f = 1$$

Limites no Infinito

$$\lim_{x \to \pm\infty} f(x)$$

Ex.1 $$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right) = \lim_{x \to \infty} (1) + \lim_{x \to \infty} \frac{1}{x} = 1 + \text{"}\cancel{\frac{1}{\infty}}\text{"} = 1$$

$\frac{1}{x} \to 0$; $\frac{1}{\infty} \to 0$



<u>Ex. 2</u> $$\lim_{x\to\infty} \frac{-2x^2+x+2}{x^2+2x-1} = \text{"}\frac{-\infty}{\infty}\text{"} \ ?$$

$$= \lim_{x\to\infty} \frac{\cancel{x^2}}{\cancel{x^2}} \cdot \frac{(-2+\overbrace{1/x}^{\to 0}+\overbrace{2/x^2}^{\to 0})}{(1+\underbrace{2/x}_{\to 0}-\underbrace{1/x^2}_{\to 0})} = -2 \ //$$

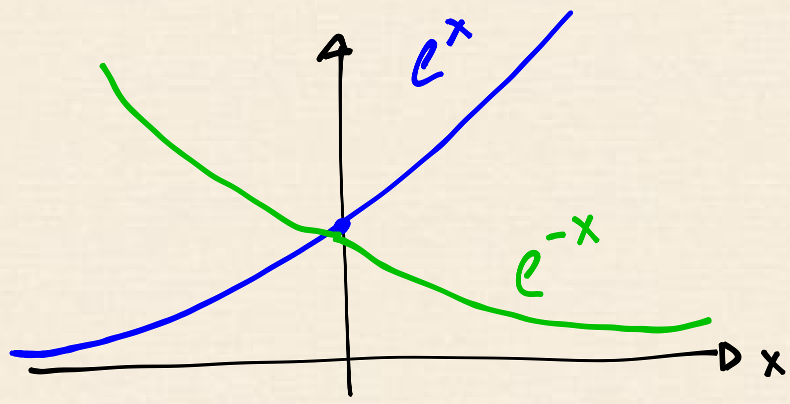
<u>Ex. 3</u> $$\lim_{x\to-\infty} \frac{x^2-4x+2}{2x^3-1} = \text{"}\frac{\infty}{-\infty}\text{"} \ ?$$

$$= \lim_{x\to-\infty} \frac{x^2}{x^3} \cdot \frac{(1-4/x+2/x^2)}{(2-1/x^3)} = \lim_{x\to-\infty} \underbrace{\frac{1}{x}}_{\to 0} \cdot \lim_{x\to-\infty} \frac{1-\overbrace{4/x}^{\to 0}+\overbrace{2/x^2}^{\to 0}}{2-\underbrace{1/x^3}_{\to 0}}$$

$$= \lim_{x\to-\infty} \frac{1}{x} \cdot \frac{1}{2} = \frac{1}{2} \underbrace{\lim_{x\to\infty} \frac{1}{x}}_{\to 0} = 0 \ //$$

- - . - - . . - -

$$\lim_{x\to\pm\infty} \frac{1}{x^n} = 0, \quad n>0$$



$$\lim_{x\to-\infty} e^x = 0$$

$$\lim_{x\to+\infty} e^{-x} = \lim_{x\to+\infty} \frac{1}{e^x} = 0$$

$$\lim_{x\to+\infty} e^x = +\infty, \quad \lim_{x\to-\infty} e^{-x} = +\infty$$

15/9/2020

LIMITES INFINITOS

Ex. 1 $f(x) = \dfrac{x}{x^2-1}$, $\lim\limits_{x \to 1^+} f$?, $\lim\limits_{x \to 1^-} f$?

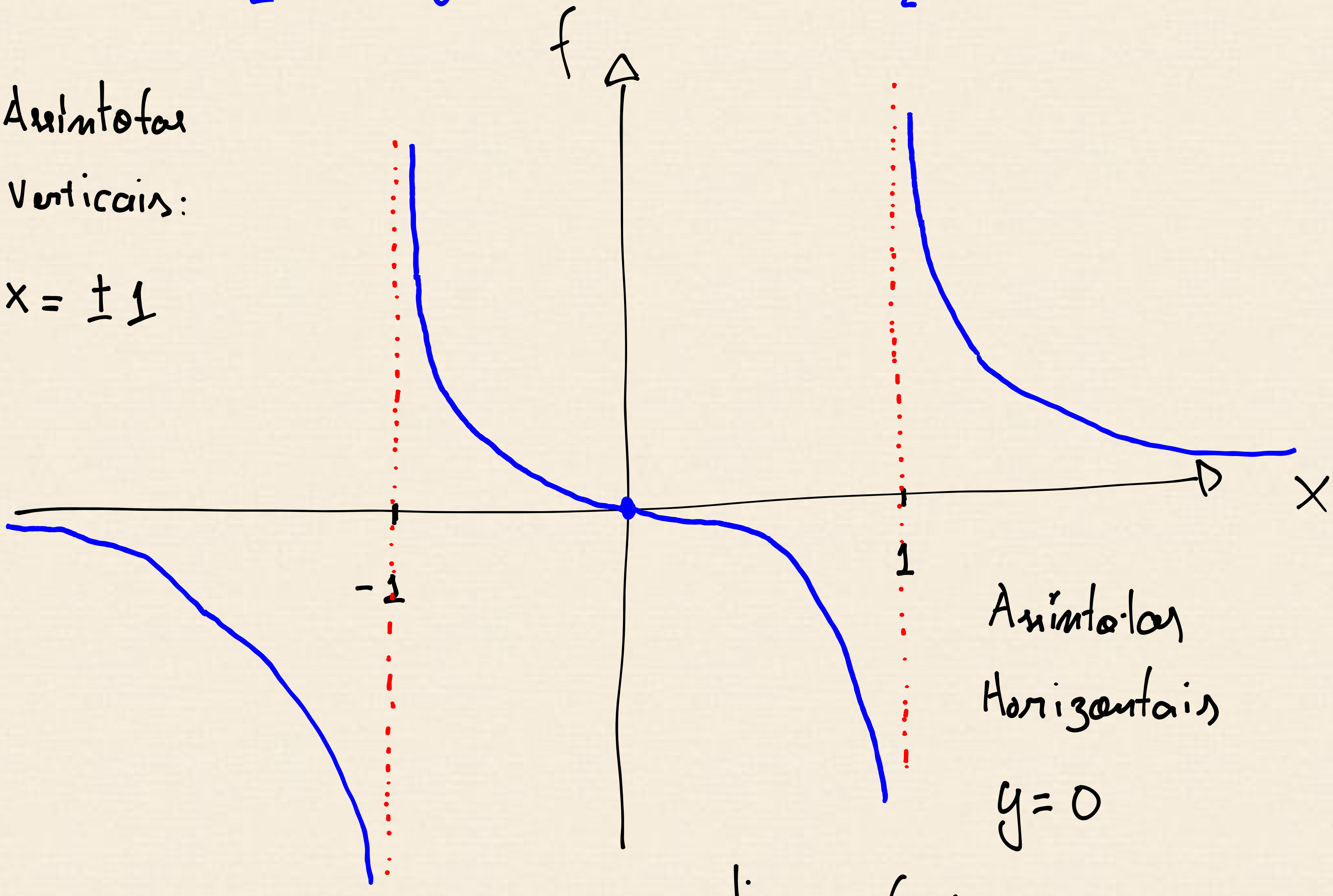
$$f(x) = \frac{x}{(x-1)(x+1)}, \quad \lim_{x \to 1^+} \frac{\overset{\to 1}{x}}{\underbrace{(x-1)}_{\to 0^+}\underbrace{(x+1)}_{\to 2}} = +\infty$$

$$\lim_{x \to 1^-} \frac{\overset{\to 1}{x}}{\underbrace{(x-1)}_{\to 0^-}\underbrace{(x+1)}_{\to 2}} = -\infty$$

$$\lim_{x \to -1^+} \frac{\overset{\to -1}{x}}{\underbrace{(x-1)}_{\to -2}\underbrace{(x+1)}_{\to 0^+}} = +\infty \; ; \; \lim_{x \to -1^-} \frac{\overset{\to -1}{x}}{\underbrace{(x-1)}_{\to -2}\underbrace{(x+1)}_{\to 0^-}} = -\infty$$

Assíntotas Verticais:

$x = \pm 1$



Assíntotas Horizontais

$y = 0$

$$\lim_{x \to \pm\infty} f(x) = 0$$

Ex. 2 $f(x) = \dfrac{x+3}{x+2}$

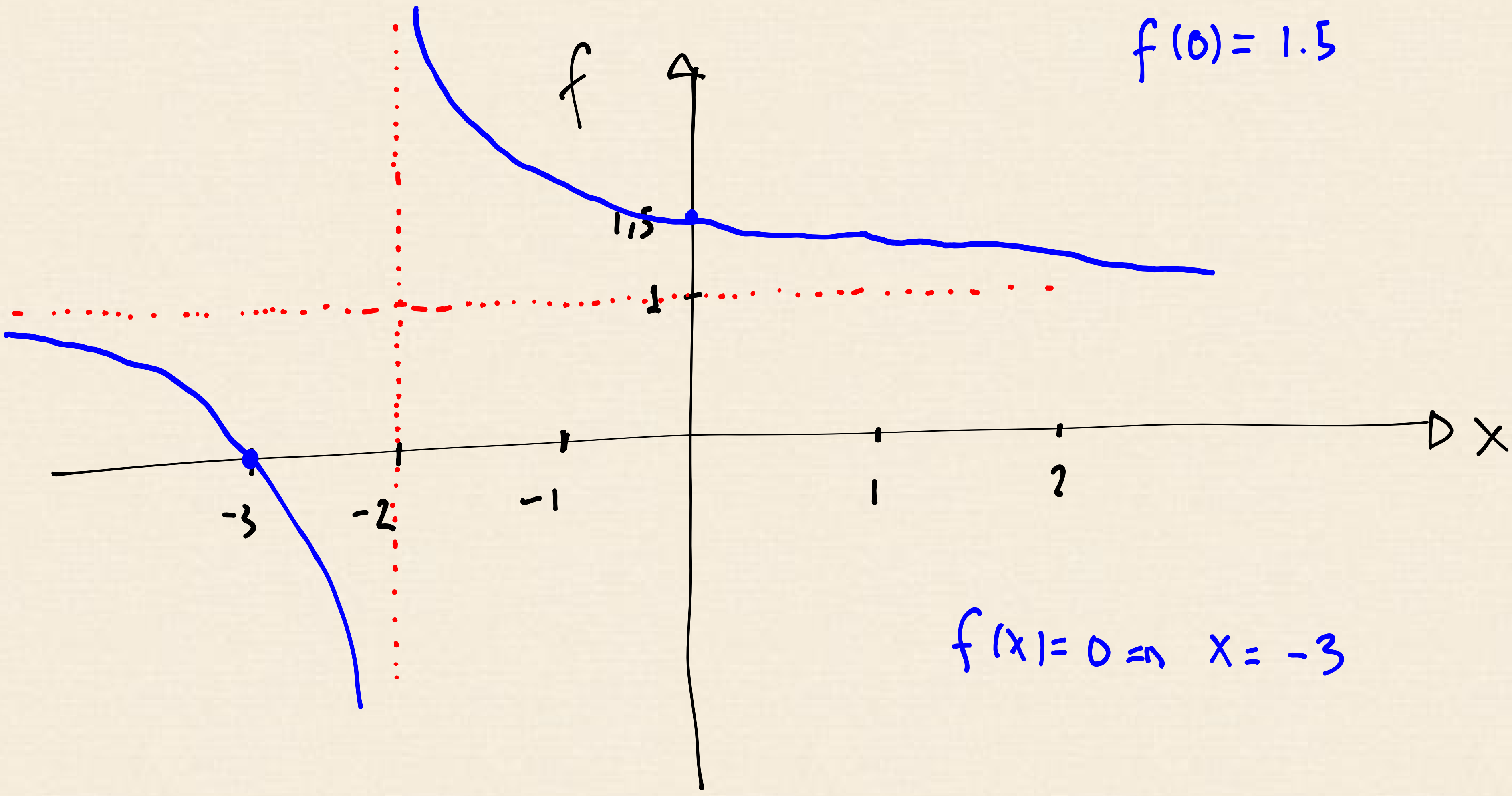
A. H. $\lim\limits_{x \to \pm\infty} f(x) = \lim\limits_{x \to \pm\infty} \dfrac{\cancel{x}\,(1 + \overbrace{3/x}^{\to 0})}{\cancel{x}\,(1 + \underbrace{2/x}_{\to 0})}$

$\lim\limits_{x \to \pm\infty} f(x) = 1$ // $\to$ $\underline{y = 1}$

A. V. $\lim\limits_{x \to a^{+,-}} f(x) = \pm\infty$

$\lim\limits_{x \to -2^-} \dfrac{x+3}{x+2} = -\infty$, $\lim\limits_{x \to -2^+} \dfrac{x+3}{x+2} = +\infty \Rightarrow \underline{x = -2}$

$f(0) = 1.5$



$f(x) = 0 \Rightarrow x = -3$

Exercícios da Lista

SEC. 1, 17 $L = \lim_{x \to 0} \dfrac{(1-\cos x)\,\text{sen}(1-\cos x)}{1 - 2\cos x + \cos^2 x} = \lim_{x \to 0} \dfrac{\cancel{(1-\cos x)}\,\text{sen}(1-\cos x)}{\cancel{(1-\cos x)}(1-\cos x)}$

$1 - 2\cos x + \cos^2 x = (1-\cos x)^2$: $u = 1-\cos x$, $x \to 0$, $u \to 0$

$L = \lim_{u \to 0} \dfrac{\text{sen}\, u}{u} = 1$ //.

SEC. 3, 3 $\lim_{x \to -\infty} \dfrac{2x+5}{\sqrt{2x^2-5}} = \lim_{x \to -\infty} \dfrac{\cancel{x}\,(2 + \overset{\to 0}{5/x})}{\cancel{x}\sqrt{2 - \underset{\to 0}{5/x^2}}} = \dfrac{2}{\sqrt{2}}$ //

$\sqrt{x^2(2 - 5/x^2)} = x\sqrt{2 - 5/x^2}$

SEC. 1, 7 $\lim_{x \to 2} \dfrac{x^4 + 2x^3 - 7x^2 - 20x - 12}{x^3 - 2x^2 - 5x + 6} = \lim_{x \to 2} \dfrac{p(x)}{q(x)} = \dfrac{p(2)}{q(2)}$

$q(2) = 8 - 8 - 10 + 6 = -4 \neq 0$

SEC. 3, 5 $\lim_{x \to -\infty} \dfrac{e^x - e^{-x}}{e^x + e^{-x}} = \lim_{x \to -\infty} \dfrac{\cancel{e^{-x}}}{\cancel{e^{-x}}} \cdot \dfrac{\overset{\to 0}{\cancel{e^{2x}}} - 1}{\underset{\to 0}{\cancel{e^{2x}}} + 1} = -1$ //.

$e^a \cdot e^b = e^{a+b}$

17/9/2020 - CONTINUIDADE

Ex.1

$$f(x) = \begin{cases} -2 & , -2 < x < -1 \\ -x^2+x & , -1 < x < 0 \\ x & , 0 \leq x < 1 \\ \sqrt{x}+1 & , x \geq 1 \end{cases}$$

$D_f = (-2,-1) \cup (-1,+\infty)$

• f é contínua em $x=-1$? $x=-1 \notin D_f$ // "Pergunta errada."

• f é contínua em $x=0$? $\Rightarrow \lim_{x\to 0} f(x) = f(0)$?

$$\left.\begin{array}{l} \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (-x^2+x) = 0 \\ \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x = 0 \end{array}\right] \Rightarrow \lim_{x\to 0} f(x) = 0$$

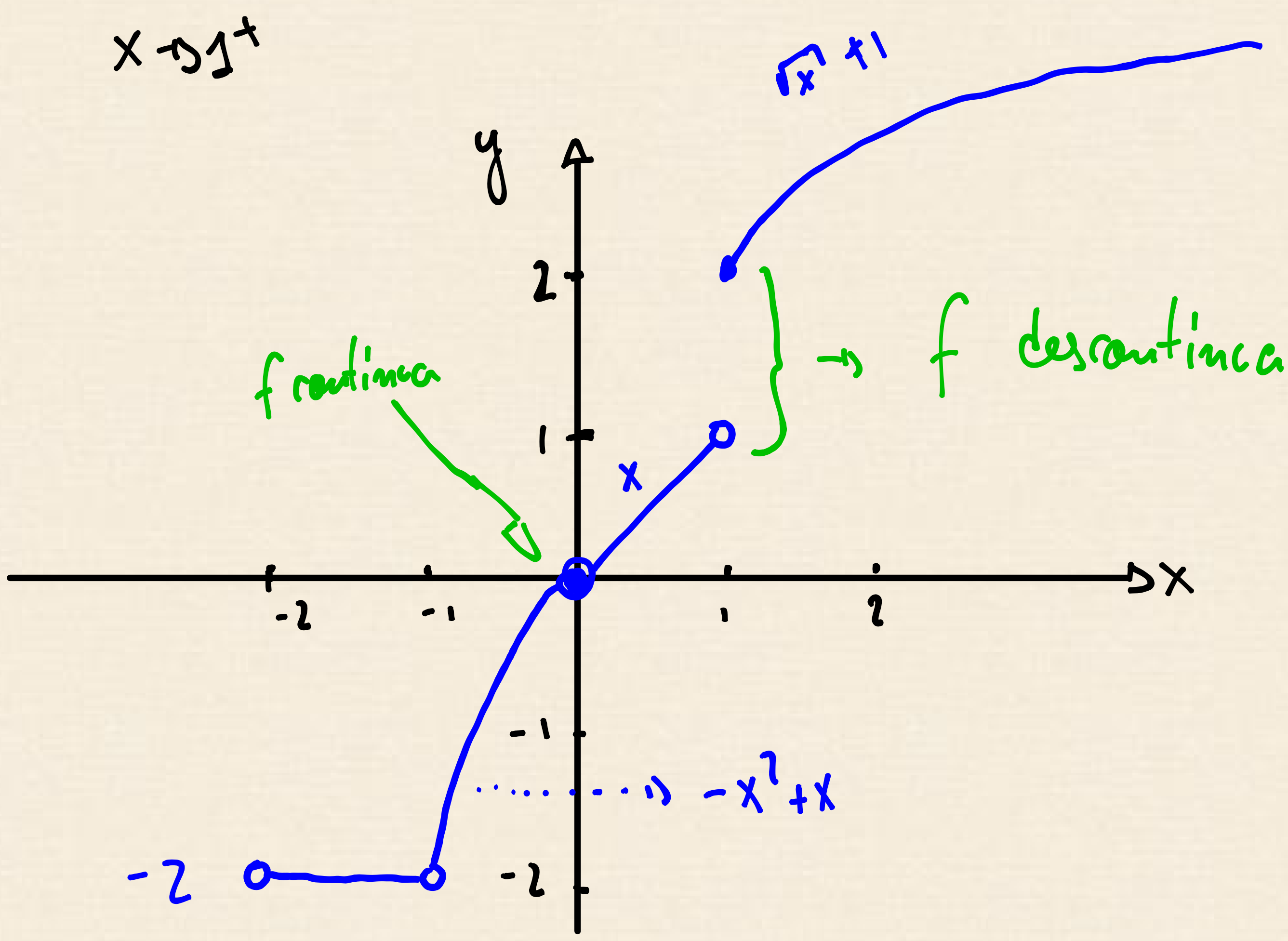
$0 = 0$

$\Rightarrow f$ é contínua em $x=0$ //

• f é contínua em $x=1$?

$\Rightarrow \lim_{x\to 1} f(x) = f(1)$? $\longrightarrow \nexists = 2 \Rightarrow f$ não é contínua em $x=1$ //

$$\left.\begin{array}{l} \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x = 1 \\ \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (\sqrt{x}+1) = 2 \end{array}\right] \Rightarrow \lim_{x\to 1} f(x) \nexists$$



Exercício SEC. 3, (7)

$$\lim_{x\to-\infty} \cos x \exp\left(\frac{x^3+2x+1}{x^2-x}\right) \ ?,$$

$$\lim_{x\to-\infty} \overbrace{\frac{x^3+2x+1}{x^2-x}}^{u} = \lim_{x\to-\infty} \frac{\cancel{x^3}}{\cancel{x^2}} \cdot \frac{1+2/x^2+1/x^3}{1-1/x}$$

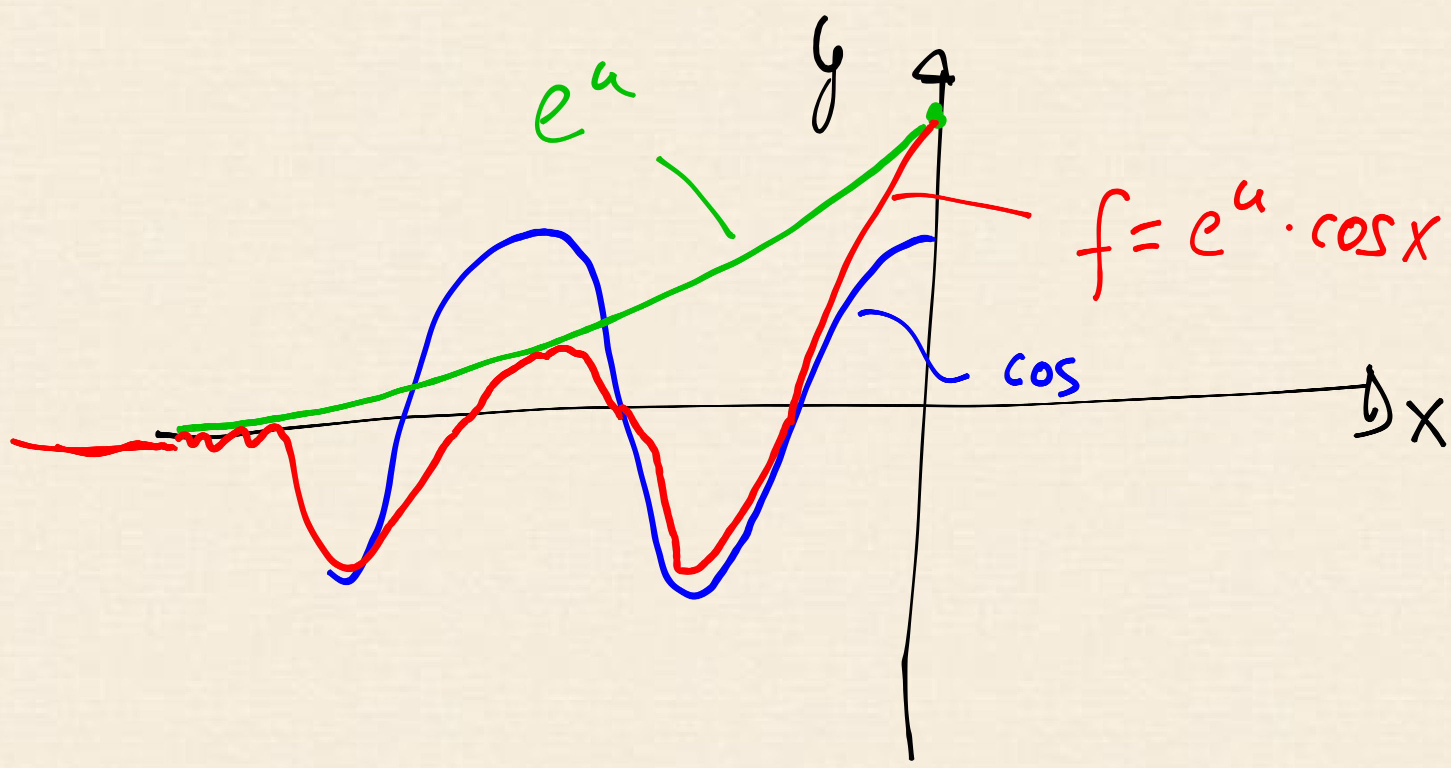
$$= \lim_{x\to-\infty} x \cdot \frac{1+2/x^2+1/x^3}{1-1/x} = -\infty$$

$$\lim_{x\to-\infty} \exp\left(\frac{x^3+2x+1}{x^2-x}\right) = \lim_{u\to-\infty} e^u$$

$$= 0 \ //$$

$$\lim_{x\to-\infty} \cos(x) \ \nexists, \text{ mas } |\cos x| \le 1$$

$$\lim_{x\to-\infty} \cos x \exp(u(x)) \propto (f \text{ limitada}) \cdot 0 = 0 \ //$$



SEC.1, 18 $L = \lim_{x \to 0} \dfrac{\cos x - 1}{x} = \text{“}\dfrac{0}{0}\text{”} \; ?$

$$\frac{\cos x - 1}{x} \cdot \frac{\cos x + 1}{\cos x + 1} = \frac{\cos^2 x - 1}{x(\cos x + 1)} = \frac{\cancel{1} - \operatorname{sen}^2 x - \cancel{1}}{x(\cos x + 1)} = -\frac{\operatorname{sen}^2 x}{x(\cos x + 1)}$$

$$= -\frac{\operatorname{sen} x}{x} \cdot \frac{\operatorname{sen} x}{\cos x + 1}$$

$$L = -\lim_{x \to 0} \underbrace{\frac{\operatorname{sen} x}{x}}_{\to 1} \cdot \lim_{x \to 0} \underbrace{\frac{\operatorname{sen} x}{\cos x + 1}}_{\to 0} = 0 \;//$$