19/11/2020

INTEGRAIS INDEFINIDAS

F é primitiva de f se

$$\frac{d}{dx} F(x) = f(x)$$

Ex. 1 $\quad f(x) = x^2$

$$\frac{d}{dx} x^3 = 3x^2 \Rightarrow \frac{d}{dx}\left(\frac{x^3}{3}\right) = x^2$$

$$F(x) = \frac{x^3}{3} + K \text{ //}$$

Ex. 2 $\quad f(x) = \cos(2x)$

$$\frac{d}{dx} \text{sen}(2x) = 2\cos(2x) \Rightarrow \frac{d}{dx}\left(\frac{\text{sen}(2x)}{2}\right) = \cos(2x)$$

$$F(x) = \frac{\text{sen}(2x)}{2} + K \text{ //}$$

Ex. 3 $\quad f(x) = \frac{2}{x}$

$$\frac{d}{dx} \ln(x) = \frac{1}{x} \Rightarrow \frac{d}{dx}\left(2\ln(x)\right) = \frac{2}{x}$$

$$F(x) = 2\ln(x) + K \text{ //}$$

$$F(x) = \ln x^2 + K \text{ //}$$

$$\frac{d}{dx} \ln x^2 = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$$

Ex. 4 $f(x) = 2x\, e^{x^2}$

$$\frac{d}{dx} e^{x^2} = 2x\, e^{x^2} \quad , \quad F(x) = e^{x^2} + K$$

Integral Indefinida

$$\int f(x)\, dx = F(x) + K$$

- - - - - - - - - - - - - -

Integral Definida $\int_a^b f(x)\, dx = F(b) - F(a)$

- - - - - - - - - - - - - - - - -

Relação da Int. Indefinida com a Derivada:

$$\frac{d}{dx}\left(F(x) + K\right) = f(x)$$

$$\frac{d}{dx}\left[\int f(x)\, dx\right] = f(x) = \int \frac{df}{\cancel{dx}} \cancel{dx} = \int df = f(x)$$

A Int. Ind. é a operação inversa da Derivada e vice-versa.

$$\boxed{\frac{d}{dx}\int f(x)\, dx = \int dx \ \frac{df}{dx}}$$

$$\int dx \ \frac{d}{dx} \sec(x) = \int \sec(x) \cdot tg(x)\, dx$$

$$\int d \sec(x) = \boxed{\sec(x) = \int \sec(x) \cdot tg(x)\, dx}$$

Ex. $\int \frac{1}{x}\,dx = \ln|x| + K$

$$\frac{d}{dx}\ln|x| = \begin{cases} \frac{d}{dx}\ln x = \frac{1}{x}, & x \geq 0 \\ \frac{d}{dx}\ln(-x) = \frac{1}{-x}(-1) = \frac{1}{x}, & x < 0 \end{cases}$$

25/11/2020

INTEGRAÇÃO POR MUDANÇA DE VARIÁVEL

$$\frac{d}{dx}F(x) = f(x)$$

$$\frac{d}{du}F(u) = f(u)$$

Pela regra da cadeia:

$$\frac{d}{dx}F(u(x)) = \frac{dF}{du}\frac{du}{dx} = f(u)\frac{du}{dx}$$

$$\int f(u)\frac{du}{dx}\,dx = \int f(u)\,du = F(u) + C$$

Ex.0

$$\int \cos(x^2)\,2x\,dx = \operatorname{sen}(x^2) + K$$

$$\frac{d}{dx}\operatorname{sen}(x^2) = 2x\cos(x^2)$$

$u = x^2$

$\frac{du}{dx} = 2x \Rightarrow du = 2x\,dx$

$$\int \cos(x^2)\,2x\,dx = \int \cos(u)\,du = \operatorname{sen}(u) + K = \operatorname{sen}(x^2) + K$$

Ex.1

$$\int \frac{2x}{x^2+1}\,dx = \int \frac{du}{u} = \ln|u| + K = \ln|x^2+1| + K$$

$u = x^2 + 1$

$du = 2x\,dx$

Ex.2 $\int \operatorname{sen}^2 x \cos x \, dx = \int u^2 \, du = \frac{u^3}{3} + K$

$u = \operatorname{sen} x$ $\qquad = \frac{\operatorname{sen}^3 x}{3} + K$ //

$du = \cos x \, dx$

Ex.3 $\int \operatorname{sen}(2x+8) \, dx = \int \operatorname{sen}(u) \frac{du}{2} = \frac{-\cos(u)}{2} + K$

$u = 2x+8$ $\qquad = \frac{-\cos(2x+8)}{2} + K$ //

$du = 2\,dx \Rightarrow dx = \frac{1}{2} du$

$\boxed{26/11/2020}$

Ex.4 $\int tg(x) \, dx = \int \frac{\operatorname{sen}(x)}{\cos(x)} dx = \int -\frac{du}{u} = -\ln|u| + K$

$u = \cos(x)$ $\qquad = -\ln|\cos(x)| + K$ //

$du = -\operatorname{sen}(x) \, dx$

Ex.5 $\int \frac{1}{(2x+8)^2} dx = \int \frac{du/2}{u^2} = \frac{1}{2} \int u^{-2} du$

$u = 2x+8$ $\qquad = \frac{1}{2} \frac{u^{-1}}{(-1)} = -\frac{1}{2} \frac{1}{u}$

$du = 2\,dx$

$= -\frac{1}{4x+16} + K$ //

Ex. 6 $\displaystyle \int x \exp\left(\frac{x^2}{3} - 1\right) dx = \int \frac{3}{2} du\, e^u = \frac{3}{2} e^u + K$

$$u = \frac{x^2}{3} - 1 \qquad\qquad = \frac{3}{2} \exp\left(\frac{x^2}{3} - 1\right) + K \quad //$$

$$du = \frac{2}{3} x\, dx$$

SEC. 1, 18 Lista: Equações Diferenciais

$$F\left(\frac{d^n}{dt^n} x(t), \frac{d^{n-1}}{dt^{n-1}} x(t), \cdots, f(x(t))\right) = 0$$

(a) $\displaystyle \frac{d}{dt} x(t) = v = \text{const.}$ $\qquad [\, F = ma = 0 \,]$

$$\int dt \frac{d}{dt} x = \int dt\, v \Rightarrow x(t) = vt + x_1 \quad //$$

(b) $\displaystyle \frac{d^2}{dt^2} x(t) = a = \text{const.}$ $\qquad [\, F = ma = \text{const.} \,]$

$$\int dt \frac{d^2}{dt^2} x = \int dt\, a \Rightarrow \int d\left(\frac{dx}{dt}\right) = at + v_1$$

$$\Rightarrow \frac{dx}{dt} = at + v_1$$

$$\Rightarrow \int dt \frac{dx}{dt} = \int dt\,(at + v_1)$$

$$x(t) = a\frac{t^2}{2} + v_1 t + x_1 \quad //$$

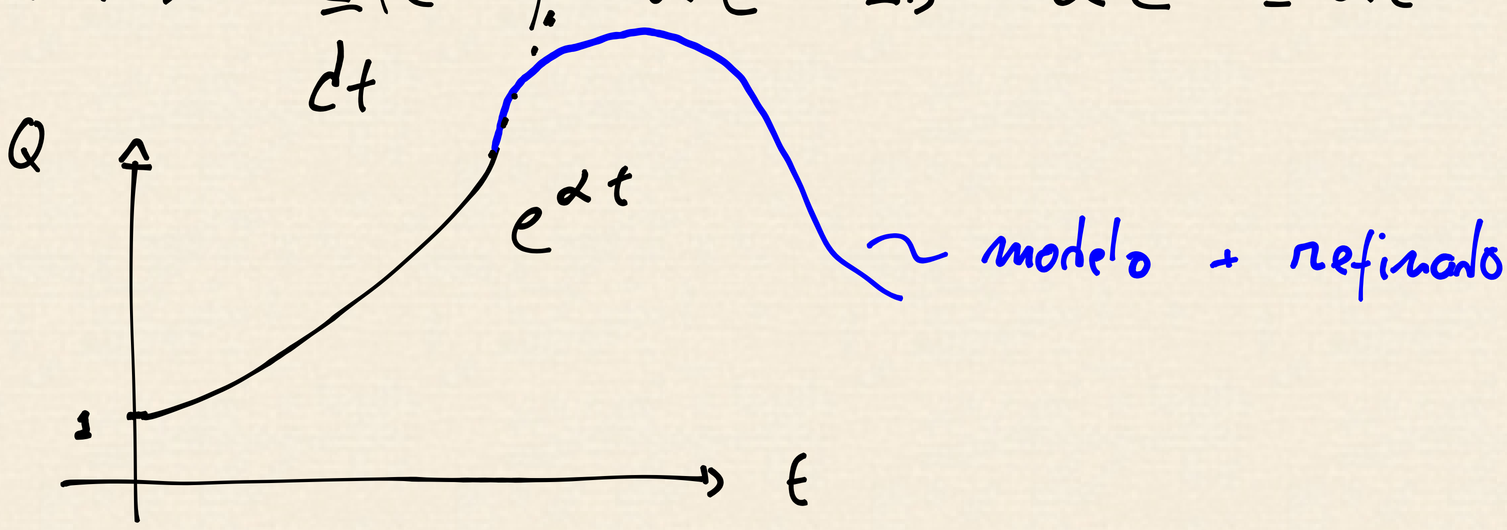
Crescimento exponencial: $\frac{d\,Q(t)}{dt} = \alpha\, Q(t)$ (1)

$$\int dt\, \frac{dQ}{dt} = \alpha \int dt\, Q(t) \Rightarrow Q(t) = \alpha \int dt\, Q(t) \quad ?$$

$$Q(t) = e^{t/t_0} \quad , \quad \frac{dQ}{dt} = \frac{1}{t_0} e^{t/t_0}$$

$\alpha = \frac{1}{t_0}$, solução é $Q(t) = e^{t\alpha}$ //

$$(1) \Rightarrow \frac{d}{dt}\left(e^{t\alpha}\right) \stackrel{!}{=} \alpha e^{t\alpha} \Rightarrow \alpha e^{t\alpha} = \alpha e^{t\alpha} \checkmark$$



INTEGRAÇÃO POR PARTES

$$f(x) = u(x) \cdot v(x)$$

$$\frac{df}{dx} = \frac{du}{dx} v + u \frac{dv}{dx}$$

$$\int dx \frac{df}{dx} = \int dx \frac{du}{dx} v + \int dx\, u \frac{dv}{dx}$$

$$\int df = \int v\, du + \int u\, dv$$

$$uv = \int v\, du + \int u\, dv$$

$$\boxed{\int u\, dv = uv - \int v\, du}$$

Ex.1 $\displaystyle \int x e^{-2x} dx = x \frac{e^{-2x}}{(-2)} - \int \frac{e^{-2x}}{(-2)} dx$

$u = x$ | $dv = e^{-2x} dx$
$du = dx$ | $v = \int e^{-2x} dx$
$v = \dfrac{e^{-2x}}{-2}$

$$= -\frac{1}{2} x e^{-2x} + \frac{1}{2} \int e^{-2x} dx$$

$$= -\frac{1}{2} x e^{-2x} + \frac{1}{2} \frac{e^{-2x}}{(-2)} + K$$

$$= \underline{\frac{e^{-2x}}{2}\left(-x - \frac{1}{2}\right) + K} = F(x)$$

$$\frac{d}{dx} F(x) = \frac{d}{dx}\left[-\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x}\right] = \cancel{-\frac{1}{2} e^{-2x}} + x e^{-2x} + \cancel{\frac{1}{2} e^{-2x}}$$

$$= x e^{-2x} \quad \checkmark$$

Ex. 2 $\int \ln(x)\,dx = x\ln x - \int \cancel{x}\cdot\frac{1}{\cancel{x}}\,dx = x\ln x - x + K$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

$\frac{d}{dx}\left[x\ln x - x\right] = \ln x + \cancel{x\cdot\frac{1}{x}} - \cancel{1} = \ln x$ ✓

Exercícios: Sec. 5, 2, Aplicação de derivadas

$$R_n(x) = \frac{1}{(n+1)!} f^{(n+1)}(z)\,(x-c)^{n+1}$$

(a) $f(x) = e^{x/\pi}$, erro no entorno de $x = \frac{\pi}{2}$ no ponto $x = \pi$

$$f(x) \simeq P_2(x) = f(\pi/2) + f'(\pi/2)(x - \pi/2) + \frac{f''(\pi/2)}{2}(x-\pi/2)^2$$

$$P_2(x) = e^{1/2} + \frac{1}{\pi}e^{1/2}(x-\pi/2) + \frac{1}{\pi^2}e^{1/2}(x-\pi/2)^2$$

$$P_2(x) = e^{1/2}\left[1 + \frac{1}{\pi}(x-\pi/2) + \frac{1}{\pi^2}(x-\pi/2)^2\right]$$

$$R_2(x) = \frac{1}{3!}f'''(z)(x-\pi/2)^3 = \frac{1}{6\pi^3}e^{3/\pi}(x-\pi/2)^3$$

Erro máximo em $x=\pi$, $R_2(\pi) = \frac{1}{6\cancel{\pi^3}}e^{3/\pi}\frac{\cancel{\pi^3}}{8} = \frac{1}{48}e^{3/\pi}$

$\frac{\pi}{2} < z < \pi \Rightarrow$ O erro máximo ocorre para $z = \pi$

$$R_2\Big|_{máx}(\pi) = \frac{1}{48}e \simeq \frac{2{,}71}{48}$$

1/12/2020

Ex. 3 (Integração por partes) $\int u\,dv = uv - \int v\,du$

$$I = \int x^2 \text{sen}(x)\,dx = -x^2\cos x - \int (-\cos x)\,2x\,dx$$

$u = x^2$; $dv = \text{sen}x\,dx$

$du = 2x\,dx$; $v = -\cos x$

$$= -x^2\cos x + \int 2x\cos x\,dx$$

$$= -x^2\cos x + \left[2x\,\text{sen}x - 2\int \text{sen}x\,dx\right]$$

$u = x$; $dv = \cos x\,dx$

$du = dx$; $v = \text{sen}x$

$$I = -x^2\cos x + 2x\,\text{sen}x + 2\cos x + K$$

$$\frac{dI}{dx} = -2x\cancel{\cos x} + x^2\,\text{sen}x + 2\cancel{\text{sen}x} + 2x\cancel{\cos x} - 2\cancel{\text{sen}x} = x^2\,\text{sen}x \checkmark$$

TÉCNICAS DE INTEGRAÇÃO

Como integrar $\int \text{sen}^n(x)\,dx$ e $\int \cos^n(x)\,dx$?

n: inteiro.

Identidade trigonométricas úteis:

$$\boxed{\text{sen}^2 x + \cos^2 x = 1}$$

$$\boxed{\text{sen}^2 x = \frac{1}{2} - \frac{\cos(2x)}{2}}, \quad \boxed{\cos^2 x = \frac{1}{2} + \frac{\cos(2x)}{2}}$$

Ex.1 $\int \cos^5 x \, dx = \int \cos^4 x \cos x \, dx = I$

$$\cos^4 x = (\cos^2 x)^2 = (1-\operatorname{sen}^2 x)^2 = 1 - 2\operatorname{sen}^2 x + \operatorname{sen}^4 x$$

$$I = \int (1 - 2\operatorname{sen}^2 x + \operatorname{sen}^4 x)\cos x \, dx$$

$$= \int \cos x \, dx - 2\int \operatorname{sen}^2 x \cos x \, dx + \int \operatorname{sen}^4 x \cos x \, dx$$

- - - - - -

$$\int \operatorname{sen}^n x \cos x \, dx = \int u^n \, du = \frac{u^{n+1}}{n+1} = \frac{\operatorname{sen}(x)^{n+1}}{n+1}$$

$u = \operatorname{sen} x$

$du = \cos x \, dx$

- - - - - - - - - - -

$$I = \operatorname{sen}(x) - \frac{2}{3}\operatorname{sen}^3(x) + \frac{\operatorname{sen}^5(x)}{5} + K$$

$$\frac{dI}{dx} = \cos x - \frac{2}{3}\cdot 3 \cdot \operatorname{sen}^2(x)\cos(x) + \frac{5}{5}\operatorname{sen}^4(x)\cos x$$

$$= \cos(x)\left[1 - 2\operatorname{sen}^2 x + \operatorname{sen}^4 x\right] = \cos^5 x \checkmark$$

Faça Ex.2 : $\int \operatorname{sen}^3 x \, dx$

Ex. 3 $\int \operatorname{sen}^4 x \, dx = I$

$$\left(\operatorname{sen}^2(x)\right)^2 = \left(\frac{1}{2} - \frac{\cos(2x)}{2}\right)^2 = \frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{\cos^2(2x)}{4}$$

$$\operatorname{sen}^4(x) = \frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{1}{4}\left[\frac{1}{2} + \frac{1}{2}\cos(4x)\right]$$

$$= \frac{3}{8} - \frac{1}{2}\cos(2x) + \frac{1}{8}\cos(4x)$$

$$\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$$

$$\Rightarrow I = \int \left(\frac{3}{8} - \frac{1}{2}\cos(2x) + \frac{1}{8}\cos(4x)\right) dx$$

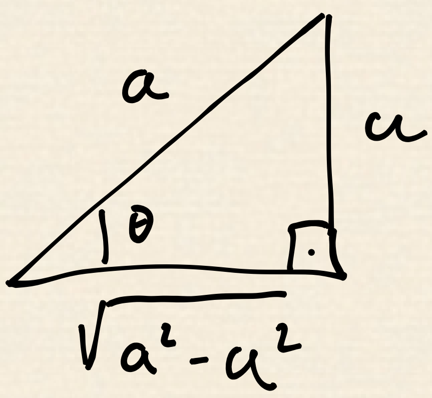
$$= \frac{3}{8}x - \frac{1}{2}\int \cos(2x)\, dx + \frac{1}{8}\int \cos(4x)\, dx$$

$$= \frac{3}{8}x - \frac{1}{2}\,\frac{\operatorname{sen}(2x)}{2} + \frac{1}{8}\,\frac{\operatorname{sen}(4x)}{4} + K$$

$$I = \frac{3}{8}x - \frac{1}{4}\operatorname{sen}(2x) + \frac{1}{32}\operatorname{sen}(4x) + K \quad //$$

$$\frac{dI}{dx} = \frac{3}{8} - \frac{1}{2}\cos(2x) + \frac{1}{8}\cos(4x) = \operatorname{sen}^4(x) \checkmark$$

SUBSTITUIÇÃO TRIGONOMÉTRICA



Quando houver funções envolvendo $\sqrt{a^2-u^2}$ numa integral, pode-se utilizar a seguinte substituição:

$$u = a \operatorname{sen}\theta$$

__Ex.1__ $\int \sqrt{4-x^2}\, dx = I$

$x = 2\operatorname{sen}\theta$, $\sqrt{4-x^2} = \sqrt{4-4\operatorname{sen}^2\theta} = 2\sqrt{1-\operatorname{sen}^2\theta}$

$dx = 2\cos\theta\, d\theta$ $\qquad\qquad = 2\cos\theta$

$$\Rightarrow I = \int 2\cos\theta\,(2\cos\theta)\, d\theta = 4\int \cos^2\theta\, d\theta$$

$$= 4\int d\theta \left[\frac{1}{2} + \frac{1}{2}\cos(2\theta)\right] = 4\left[\frac{\theta}{2} + \frac{1}{2}\frac{\operatorname{sen}(2\theta)}{2}\right] + K$$

$\Rightarrow I = 2\theta + \operatorname{sen}(2\theta) + K$, $x = 2\operatorname{sen}\theta$

$\Rightarrow \theta = \operatorname{arcsen}(x/2)$

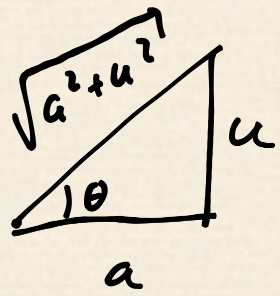
$$I = 2\operatorname{arcsen}(x/2) + \operatorname{sen}(2\operatorname{arcsen}(x/2)) + K$$

. - - - - - - . -

$\int \sqrt{u}\, dx = \int \sqrt{u}\, \frac{du}{-2x} = \int \frac{\sqrt{u}}{-2\sqrt{1-u}}\, du$?

$u = 1-x^2$

$du = -2x\, dx$ $\Rightarrow x^2 = 1-u \Rightarrow |x| = \sqrt{1-u}$



Ex.2

$$\int \sqrt{9+x^2}\, dx = I$$

$$u = a\,tg(\theta)$$

$$\frac{d}{d\theta} tg\theta = \frac{d}{d\theta}\left(\frac{sen\theta}{cos\theta}\right)$$

$$= \frac{cos^2\theta + sen^2\theta}{cos^2\theta} = \frac{1}{cos^2\theta}$$

$$\frac{sen^2\theta + cos^2\theta}{cos^2\theta} = \frac{1}{cos^2\theta}$$

$$tg^2\theta + 1 = \frac{1}{cos^2\theta}$$

$$x = 3\,tg\theta \Rightarrow \sqrt{9+x^2} = 3\sqrt{1+tg^2\theta}$$

$$dx = 3\frac{1}{cos^2\theta}d\theta \qquad \boxed{\sqrt{9+x^2} = 3\frac{1}{cos\theta}}$$

$$I = \int \frac{3}{cos\theta} \cdot \frac{3}{cos^2\theta}\, d\theta$$

$$I = 9\int \frac{d\theta}{cos^3\theta} \quad ?$$

$$u = cos\theta$$

$$du = -sen\theta\, d\theta$$

$$I = 9\int \frac{d\theta}{cos^2\theta\, cos\theta} = 9\int \frac{d\theta}{(1-sen^2\theta)\,cos\theta} \quad ?$$

FRAÇÕES PARCIAIS

$$\frac{mx+n}{(x-\alpha)(x-\beta)} = \frac{A}{x-\alpha} + \frac{B}{x-\beta} = \frac{A(x-\beta)+B(x-\alpha)}{(x-\alpha)(x-\beta)}$$

$$= \frac{(A+B)x - (A\beta + B\alpha)}{(x-\alpha)(x-\beta)}$$

Ex.1

$$\int \frac{1}{x^2-1}\, dx = \int \frac{dx}{(x-1)(x+1)} = \int dx\left[\frac{A}{x-1} + \frac{B}{x+1}\right]$$

$+0x$

$$= \int dx\left[\frac{(A+B)x + A - B}{(x-1)(x+1)}\right].$$

$$\begin{cases} A+B=0 \Rightarrow A=-B \Rightarrow B=-1/2 \\ A-B=1 \Rightarrow 2A=1 \Rightarrow A=1/2 \end{cases}$$

$$\frac{1}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{1/2}{x-1} - \frac{1/2}{x+1} = \frac{\cancel{1/2 x} + 1/2 - \cancel{1/2 x} + 1/2}{(x-1)(x+1)}$$

$A = 1/2 \quad , B = -1/2$

$$= \frac{1}{x^2-1}.$$

$$\int \frac{dx}{x^2-1} = \int dx \left[\frac{1/2}{x-1} - \frac{1/2}{x+1} \right] = I$$

$$\int \frac{dx}{ax+b} = \int \frac{1}{a} \frac{du}{u} = \frac{1}{a} \ln|u| = \frac{1}{a} \ln|ax+b|$$

$u = ax+b$

$du = a\,dx$

$$I = \frac{1}{2} \ln|x-1| - \frac{1}{2} \ln|x+1| + K //$$

$$I = \frac{1}{2} \ln\left(\frac{|x-1|}{|x+1|}\right) + K = \ln\left(\sqrt{\frac{|x-1|}{|x+1|}}\right) + K //$$

Ex.2 $\displaystyle\int \frac{x}{x^2+x-2}\,dx = I$

$$x^2+x-2 = (x-\alpha)(x-\beta) = (x-1)(x+2)$$

$$x^2+x-2=0 \Rightarrow x = \frac{1}{2}\left[-1 \pm\sqrt{1+8}\right] \Rightarrow \alpha = 1$$
$$\beta = -2$$

$$\frac{1x+0}{x^2+x-2} = \frac{A}{x-1} + \frac{B}{x+2} = \frac{Ax+2A+Bx-B}{(x-1)(x+2)} = \frac{(A+B)x+2A-B}{(x-1)(x+2)}$$

$$\Rightarrow \begin{cases} A+B=1 \Rightarrow 3A=1 \Rightarrow A=1/3 \\ 2A-B=0 \Rightarrow B=2A \Rightarrow B=2/3 \end{cases}$$

$$\frac{x}{x^2+x-2} = \frac{1/3}{x-1} + \frac{2/3}{x+2} = \frac{\frac{1}{3}x+\cancel{\frac{2}{3}}+\frac{2}{3}x-\cancel{\frac{2}{3}}}{(x+2)(x-1)} \checkmark$$

$$\Rightarrow I = \int dx\left[\frac{1/3}{x-1} + \frac{2/3}{x+2}\right] = \frac{1}{3}\ln|x-1| + \frac{2}{3}\ln|x+2| + K \;//$$

- - - - - - - - -

Outros casos...

$$\frac{mx+n}{(x-\alpha)^2} = \frac{A}{x-\alpha} + \frac{B}{(x-\alpha)^2}$$

$$\frac{mx^2+nx+p}{(x-\alpha)(x-\beta)(x-\gamma)} = \frac{A}{x-\alpha} + \frac{B}{x-\beta} + \frac{C}{x-\gamma}$$

8/12/2020

EXERCÍCIOS DE INTEGRAIS INDEFINIDAS

SEC. 1, 8

$$\int \frac{x}{2}\, sen(3x^2+2)\,dx = \int \frac{1}{2}\cdot\frac{du}{6}\, sen(u) = \frac{-1}{12}\cos(u) + K$$

$u = 3x^2+2$

$du = 6x\, dx \rightarrow x\,dx = \frac{1}{6}du$

$$= -\frac{1}{12}\cos(3x^2+2) + K$$

13 $$\int (e^{2t}+2)^{1/3} e^{2t}\,dt = \int u^{1/3}\,\frac{du}{2} = \frac{u^{4/3}}{\frac{4}{3}\cdot 2} + K$$

$u = e^{2t}+2$

$du = 2e^{2t}\,dt$

$$= \frac{3}{8}(e^{2t}+2)^{4/3} + K$$

15 Tentar, mas é extenso.

$u = \sqrt{x+3} \rightarrow du = \frac{1}{2}\frac{1}{\sqrt{x+3}}dx = \frac{1}{2}\frac{dx}{u}$

$$\int \frac{\sqrt{x+3}}{x-1}dx = \int \frac{u\cdot 2u\,du}{x-1}$$

$u^2 = x+3$

$x = u^2-3$

$x-1 = u^2-4$

$$= \int \frac{2u^2}{u^2-4}du$$

$$= \int \frac{2u^2}{(u-2)(u+2)}du = \int du\left[\frac{A}{u-2} + \frac{B}{u+2} + C\right] = \ldots$$

SEC. 2, (1) $\int x e^{-x} dx = x e^{-x} + \int e^{-x} dx$

$u = x$; $dv = e^{-x} dx$

$du = dx$; $v = \frac{e^{-x}}{(-1)} = -e^{-x}$

$= x e^{-x} - e^{-x} + K$

$= e^{-x}(x - 1) + K$

(9) $\int x^{-3} e^{1/x} dx = \int x^{-3} e^{u} (-1) x^2 du$

$u = x^{-1} = 1/x$

$du = -x^{-2} dx$

$dx = -x^2 du$

$= -\int x^{-1} e^{u} du = -\int u e^{u} du = I$

$\int w dv = wv - \int v dw$

$w = u$; $dv = e^{u} du$

$dw = du$; $v = e^{u}$

$\Rightarrow I = -u e^{u} + \int e^{u} du = e^{u}(1 - u) + K$

$I = e^{1/x}\left(1 - \frac{1}{x}\right) + K$

(5) $I = \int \text{sen}^3(x)\, dx = -\text{sen}^2 x \cos x + \underbrace{\int \cos x\,(2 \,\text{sen} x \cos x)\, dx}_{I_2}$

$u = \text{sen}^2(x)$; $dv = \text{sen} x\, dx$

$du = 2\, \text{sen} x \cos x\, dx$; $v = -\cos x$

$I = -\text{sen}^2 \cos x - \frac{2}{3} \cos^3 x + K$

$I_2 = 2 \int \cos^2 x\, \text{sen} x\, dx = -2 \int u^2 du$

$u = \cos x$

$du = -\text{sen} x\, dx$

$I_2 = -2 \frac{u^3}{3} = -\frac{2}{3} \cos^3 x$

Sec. 3, 1.a $\int \operatorname{sen}^3 x \, dx = \int (1-\cos^2 x) \operatorname{sen} x \, dx$

$\operatorname{sen}^3 x = \operatorname{sen}^2 x \operatorname{sen} x$
$= (1-\cos^2 x) \operatorname{sen} x$

$$= \int \operatorname{sen} x \, dx - \int \cos^2 x \operatorname{sen} x \, dx$$

$$= -\cos x + \frac{1}{3}\cos^3 x + K$$

$$I_1 = -\operatorname{sen}^2 x \cos x - \frac{2}{3}\cos^3 x + K_1 = \cos x\left[-\operatorname{sen}^2 x - \frac{2}{3}\cos^2 x\right] + K_1$$

$$I_2 = -\cos x + \frac{1}{3}\cos^3 x + K_2 = \cos x\left[-1 + \frac{1}{3}\cos^2 x\right] + K_2$$

$$I_1 = \cos x\left[-1 + \cos^2 x - \frac{2}{3}\cos^2 x\right] + K_1$$

$$= \cos x\left[-1 + \frac{1}{3}\cos^2 x\right] + K_1$$

---

2. Substituição trigonométrica (desconsiderar $\sqrt{a^2+u^2}$)

(a) $\int \sqrt{25-4x^2}\, dx = 5\int \sqrt{1 - \frac{4}{25}x^2}\, dx = 5\int \sqrt{1 - \underbrace{\frac{4}{25}a^2}_{=1}\operatorname{sen}^2\theta}\, dx$

$u = a\operatorname{sen}\theta \rightarrow x = \frac{5}{2}\operatorname{sen}\theta$

$du = a\cos\theta \, d\theta \rightarrow dx = \frac{5}{2}\cos\theta \, d\theta$

$$= 5\int \sqrt{1-\operatorname{sen}^2\theta} \cdot \frac{5}{2}\cos\theta \, d\theta$$

$$= \frac{25}{2}\int |\cos\theta| \cos\theta \, d\theta = I$$

$$I = \pm\frac{25}{2}\int \cos^2\theta \, d\theta = \pm\frac{25}{2}\int \left[\frac{1}{2} + \frac{\cos(2\theta)}{2}\right] d\theta$$

$$I = \pm\frac{25}{2}\left[\frac{\theta}{2} + \frac{\operatorname{sen}(2\theta)}{4}\right] + K, \text{ com } \theta = \operatorname{arcsen}\left(\frac{2x}{5}\right).$$

③ (d) $\int \frac{x+2}{x^2+2x-3}\,dx = I$

$$x^2+2x-3=0$$

$$x = \frac{1}{2}\left[-2 \pm \sqrt{4+12}\right]$$

$$x_1 = 1, \quad x_2 = -3$$

$$x^2+2x-3 = (x-1)(x+3)$$

$$\frac{x+2}{x^2+2x-3} = \frac{x+2}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$= \frac{Ax+3A+Bx-B}{(x-1)(x+3)}$$

$$= \frac{(A+B)x+3A-B}{(x-1)(x+3)}$$

$$\Rightarrow \begin{cases} A+B=1 \rightarrow B = 1-A = 1/4 \\ 3A-B=2 \rightarrow 3A-1+A=2 \end{cases}$$

$$\Rightarrow A = 3/4$$

$$\Rightarrow \frac{x+2}{x^2+2x-3} = \frac{3/4}{x-1} + \frac{1/4}{x+3} = \frac{\frac{3}{4}x + \frac{9}{4} + \frac{1}{4}x - \frac{1}{4}}{(x-1)(x+3)} \checkmark$$

$$I = \int dx \left[\frac{3/4}{x-1} + \frac{1/4}{x+3}\right] = \frac{3}{4}\ln|x-1| + \frac{1}{4}\ln|x+3| + K$$