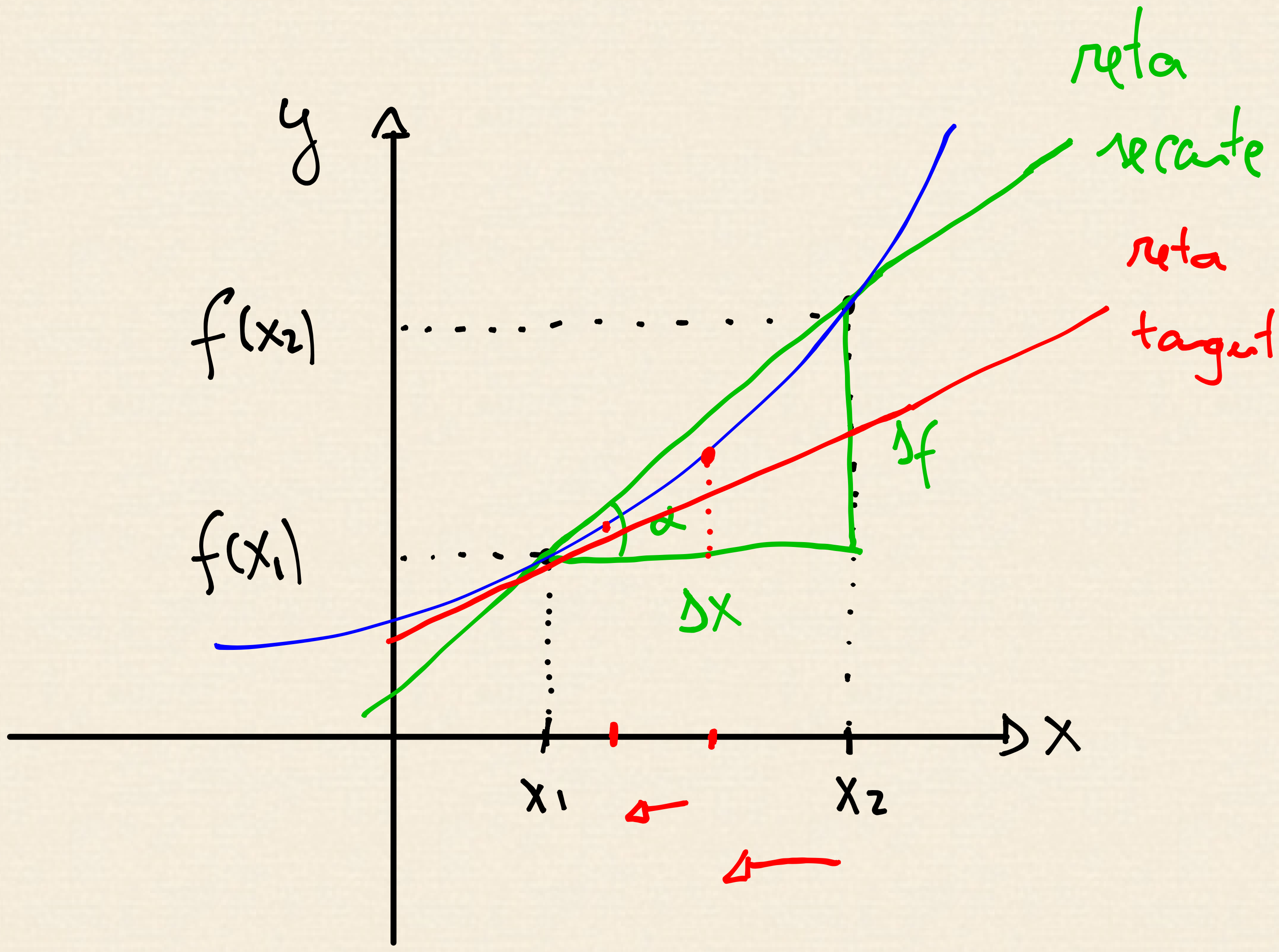
# 22/9/2020 DERIVADAS

$tg(\alpha) = \dfrac{\Delta f}{\Delta x}$




Reta tangente no ponto $X_1$ tem $tg(\alpha)$ : $\lim\limits_{\Delta x \to 0} \dfrac{f(x_1+\Delta x) - f(x_1)}{\Delta x} = \dfrac{df(x_1)}{dx}$

Para qualquer $x \in D_f$ : $\dfrac{df(x)}{dx} = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

Ex.1 $f(x) = x + 2$

Pela geometria: coeficiente angular $= 1 = tg\alpha \Rightarrow \alpha = 45°$ //

Derivada em $x = 1$ : $\lim\limits_{h \to 0} \dfrac{f(1+h) - f(1)}{h} = f'(1)$

$f'(1) = \lim\limits_{h \to 0} \dfrac{(\cancel{1}+h) + \cancel{2} - \cancel{3}}{h} = \lim\limits_{h \to 0} \dfrac{\cancel{h}}{\cancel{h}} = 1 = tg\alpha$ //

$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} \dfrac{\cancel{x}+h+\cancel{2}-\cancel{x}-\cancel{2}}{h} = \lim\limits_{h \to 0} \dfrac{\cancel{h}}{\cancel{h}} = 1$ //

Ex.2 $f(x) = x^2 + 2x$ , $f'(-1) = ?$

$$f'(-1) = \lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h} = \underbrace{\lim_{h \to 0} \frac{(-1+h)^2 + 2(-1+h) - 1 + 2}{h}}_{⊛}$$

$$⊛ = \frac{1}{h}\left[\cancel{1} - \cancel{2h} + h^2 - \cancel{2} + \cancel{2h} - \cancel{1} + \cancel{2}\right] = \frac{h^2}{h} = h$$

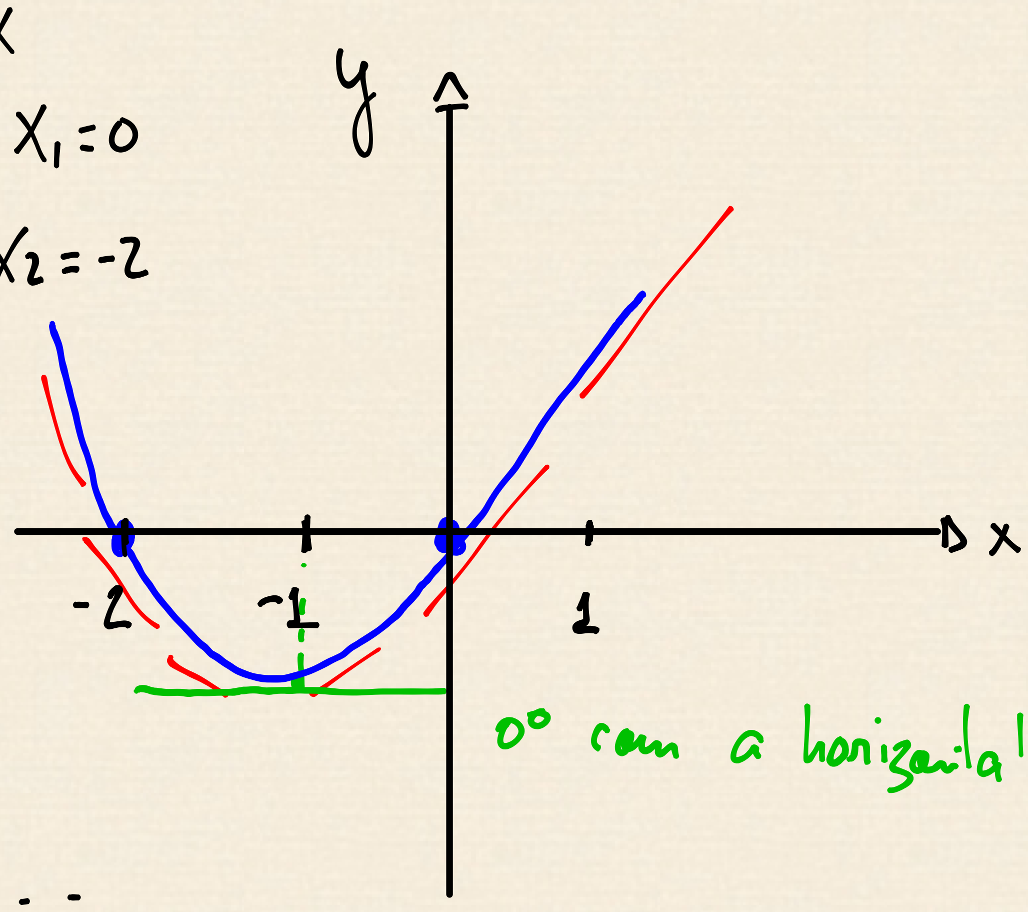
$$f'(-1) = \lim_{h \to 0} h = 0 = tg(\alpha) \Rightarrow \alpha = 0° \; //$$

Vértice da parábola? $x_v = -\frac{b}{2a} = -\frac{2}{2} = -1 \; //$

$f(x) = 0 = x^2 + 2x$

$x(x+2) = 0 \to x_1 = 0$

$\to x_2 = -2$




. . . . . . . . . . . . . .

Caso geral $f(x) = ax^2 + bx + c$ , $f'(x) = 0$ ?

$$f'(x) = \lim_{h \to 0} \frac{\overbrace{f(x+h) - f(x)}^{F}}{h}$$

$$F = a(x+h)^2 + b(\cancel{x}+h) + \cancel{c} - ax^2 - \cancel{bx} - \cancel{c}$$

$$F = a(\cancel{x^2} + 2hx + h^2) + bh - \cancel{ax^2} = h^2 a + 2ahx + bh$$

$$f'(x) = \lim_{h \to 0} \frac{h^2 a + 2ahx + bh}{h} = \lim_{h \to 0} \left[ ha + 2ax + b \right]$$

$$\boxed{f'(x) = 2ax + b} \qquad f'(x) = 0 \, ?$$

$$2ax + b = 0 \Rightarrow X_v = \frac{-b}{2a}$$

---

24/9/2020

A derivada de $f(x)$ é $\frac{df(x)}{dx}$ ou $f'(x)$.

$$\frac{df}{dx} = \begin{cases} \text{a variação de } f(x) \\ tg(\alpha) \text{ da reta tangente de } f \text{ em } X. \end{cases}$$

Sobre a existência da derivada.

Ex.1 $f(x) = |x-1|$, mostre que $f'(1) \nexists$.

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}, \quad f(x) = \begin{cases} x-1, & x-1 \geq 0 \\ -(x-1), & x-1 < 0 \end{cases}$$

O limite bilateral deve ser computado com os limites laterais.

$$\lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0^+} \frac{|1+h-1| - |1-1|}{h} = \lim_{h \to 0^+} \frac{|h|}{h}$$

$$= \lim_{h \to 0^+} \frac{h}{h} = 1$$

$$\lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0^-} \frac{|h|}{h} = \lim_{h \to 0^-} \frac{-\cancel{h}}{\cancel{h}} = -1 \,//$$

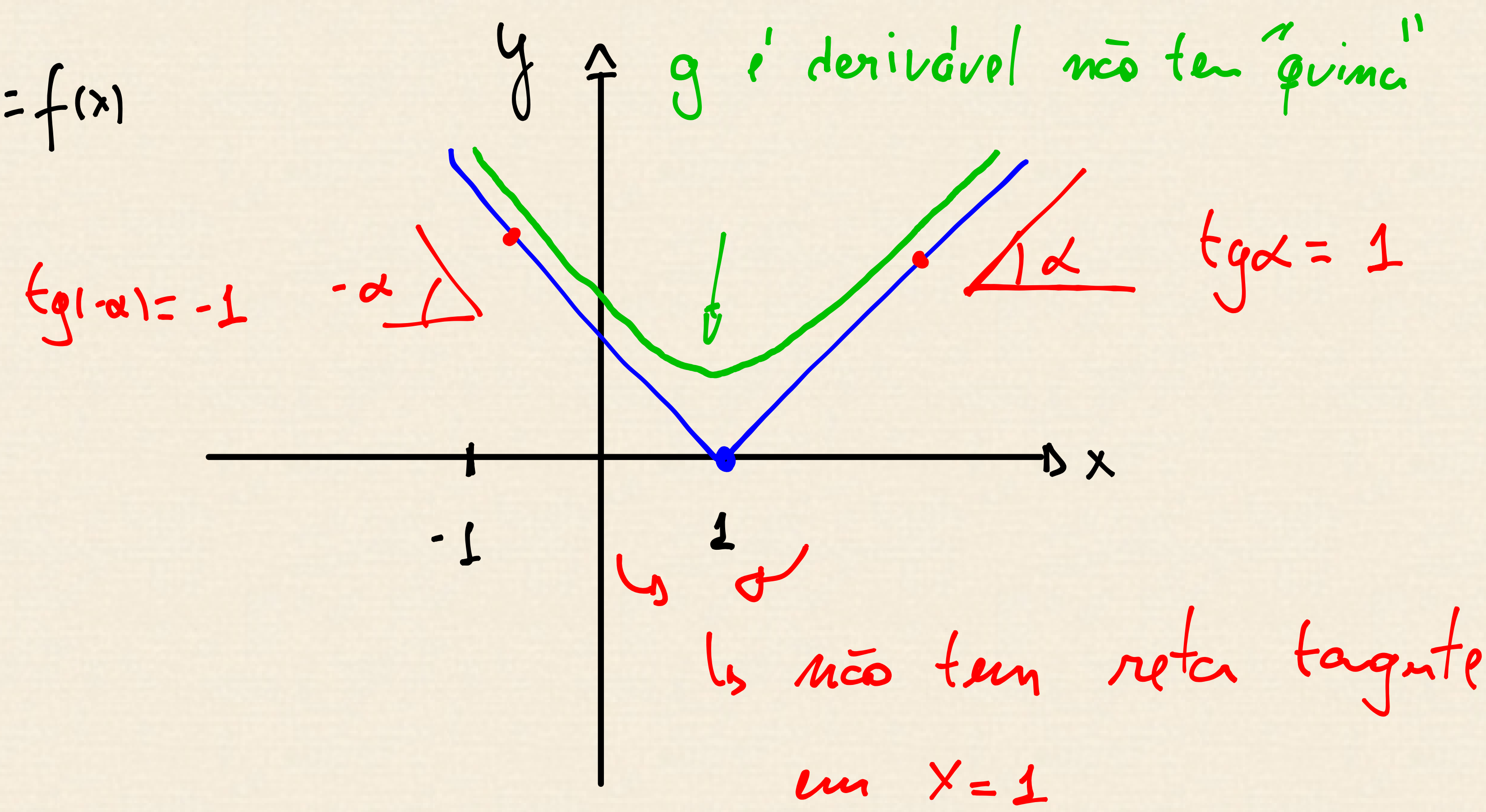
$$\lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h} \neq \lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h}$$

TEO. EXISTÊNCIA DO LIMITE $\Rightarrow \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} \nexists$.

Então $f'(1) \nexists$.

Quando uma função não tem derivada em algum $x \in D_f$, dizemos que é <u>não derivável</u>.

$|x-1| = f(x)$



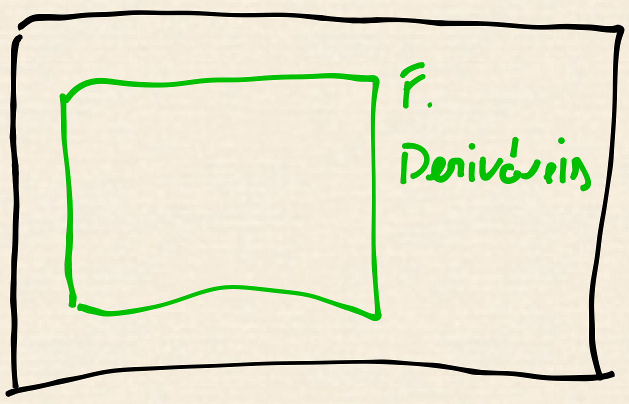

$$f'(x \mid x<1) = \lim_{h \to 0} \frac{-(\cancel{x}+h-\cancel{1}) + (\cancel{x}-\cancel{1})}{h} = -1 \,//$$

$$f'(x \mid x \geq 1) = \lim_{h \to 0} \frac{\cancel{x}+h-\cancel{1} - (\cancel{x}-\cancel{1})}{h} = 1 \,//$$

$f$ é contínua, mas não derivável.

$\lim_{x \to c} f(x) = f(c)$ para $\forall\ c \in D_f$, $f$ é contínua.

TEO.: Uma função derivável é contínua.




F. Contínuas

Derivada de função potência:

$$f(x) = x^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$\frac{(x+h)^n - x^n}{h} = \frac{\left(\cancel{x^n} + \overset{n}{\alpha} x^{n-1} \cancel{h} + \beta x^{n-2} \overset{h + \gamma h^2 + \dots}{\cancel{h^2}} + \dots\right) - \cancel{x^n}}{\cancel{h}}$$

$$\lim_{h \to 0} \frac{(x+h)^n - x^n}{h} = \lim_{h \to 0} n x^{n-1} + \overset{\to 0}{\cancel{\beta x^{n-2} h}} + \overset{\to 0}{\cancel{\gamma x^{n-3} h^2}} + \dots$$

$$= n x^{n-1}$$

$$\boxed{\frac{d}{dx} x^n = n x^{n-1}}$$

29/9/2020

# Regras de Derivação

① $f(x) = c\,g(x)$, $c = \text{const.}$

$$\frac{df}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{c\,g(x+h) - c\,g(x)}{h}$$

$$= c \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} = c\,\frac{dg}{dx}$$

$$\boxed{f' = c\,g'}$$

② $f(x) = g(x) + h(x)$

$$f' = \lim_{\alpha\to 0} \frac{g(x+\alpha) + h(x+\alpha) - g(x) - h(x)}{\alpha}$$

$$= \lim_{\alpha\to 0} \left[ \frac{g(x+\alpha) - g(x)}{\alpha} + \frac{h(x+\alpha) - h(x)}{\alpha} \right]$$

$$\boxed{f' = g' + h'}$$

③ $f(x) = g(x) \cdot h(x)$, $\boxed{f' = g' \cdot h + h' \cdot g}$

④ $f(x) = \dfrac{g(x)}{h(x)}$, $\boxed{f' = \dfrac{g' \cdot h - h' \cdot g}{h^2}}$

Derivadas de funções:

① $f(x) = c = \text{const.}$, $f' = 0$

② $f(x) = x^n$, $f' = n x^{n-1}$

③ $f(x) = \sum_{k=0}^{n} a_k x^k = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n$

$$\boxed{f' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots n a_n x^{n-1}}$$

Ex.1 $f(x) = x^3 + 2x - 1$

$f'(x) = 3x^2 + 2$

fazer pela regra do quociente.

Ex.2 $f(x) = (3x^4 + 2x^2) x^{-2} = \dfrac{3x^4 + 2x^2}{x^2}$

$f = g \cdot h$

$f' = (3x^4 + 2x^2)' x^{-2} + (3x^4 + 2x^2)(x^{-2})'$ - pela regra do produto

$f' = (12x^3 + 4x) x^{-2} + (3x^4 + 2x^2)(-2) x^{-3}$

$f' = 12x + \cancel{4x^{-1}} - 6x - \cancel{4x^{-1}} \Rightarrow f'(x) = 6x$ //

$f = 3x^2 + 2$, $f' = 6x$ // - como função polinomial

Ex.3

$f(x) = \dfrac{x^3 - 2x^2}{3x - 8}$, $f' = \dfrac{(x^3 - 2x^2)' \cdot (3x - 8) - (x^3 - 2x^2) \cdot (3x - 8)'}{(3x - 8)^2}$

$$f' = \frac{(3x^2 - 4x)(3x - 8) - (x^3 - 2x^2) \cdot 3}{(3x - 8)^2}$$

$$f' = \frac{(3x^2-4x)(3x-8) - (x^3-2x^2)\cdot 3}{(3x-8)^2}$$

$$f' = \frac{3x^2-4x}{3x-8} - \frac{3(x^3-2x^2)}{(3x-8)^2} //$$

$$f' = (9x^3 - 12x^2 - 24x^2 + 32x - 3x^3 + 6x^2) / (3x-8)^2$$

$$f' = (6x^3 - 30x^2 + 32x) / (3x-8)^2 //$$

- - - - - - - - - - -

Exercícios Sec. 7

② $f(x) = (x^2+2)(3-x)$

$$f' = (x^2+2)'(3-x) + (x^2+2)(3-x)' = 2x(3-x) + (x^2+2)(-1)$$

$$f' = 6x - 2x^2 - x^2 - 2 \Rightarrow f' = -3x^2 + 6x - 2 //$$

③ $f(x) = (x^3 - x^2 + 3)\, x^{-5}$

$$f(x) = x^{-2} - x^{-3} + 3x^{-5}$$

$$f'(x) = -2x^{-3} + 3x^{-4} - 15x^{-6} //$$

⑤ $f(x) = \frac{4-x}{5-x^2}$, $f' = \frac{(4-x)'(5-x^2) - (4-x)(5-x^2)'}{(5-x^2)^2}$

$$f' = \frac{(-1)(5-x^2) - (4-x)(-2x)}{(5-x^2)^2} = \frac{-5 + x^2 + 8x - 2x^2}{(5-x^2)^2}$$

$$f' = \frac{-x^2 + 8x - 5}{(5-x^2)^2} //$$

1/10/2020

# REGRA DA CADEIA

$$y = g(u) \,,\quad u = f(x) \Rightarrow y(x) = g(f(x))$$

função composta

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}}$$

Ex. 1

$$f(x) = (x^3 + 2x - 1)^3 = u^3_{(x)} \,,\quad u(x) = x^3 + 2x - 1$$

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx} \,,\quad \frac{df}{du} = \frac{d}{du}u^3 = 3u^2$$

$$\frac{du}{dx} = \frac{d}{dx}(x^3 + 2x - 1) = 3x^2 + 2$$

$$\frac{df}{dx} = 3u^2(3x^2+2) \Rightarrow \frac{df}{dx} = 3(x^3+2x-1)^2\,(3x^2+2) \quad //.$$

Ex. 2

$$f(x) = \frac{g(x)}{h(x)} = g(x)\,h^{-1}(x)$$

$$\frac{df}{dx} = \frac{dg}{dx}h^{-1}(x) + g(x)\underbrace{\frac{d}{dx}h^{-1}(x)}$$

$$= \frac{d(h^{-1})}{dh}\frac{dh}{dx} = -h^{-2}\frac{dh}{dx}$$

$$\frac{df}{dx} = \frac{dg}{dx}h^{-1} - g(x)\,h^{-2}\frac{dh}{dx} = \left(\frac{dg}{dx}\cdot h - g\frac{dh}{dx}\right)\Big/ h^2$$

Ex.3 $f(x) = \frac{3x+2}{2x+1} = (3x+2)(2x+1)^{-1}$

$$\frac{df}{dx} = \left[\frac{d}{dx}(3x+2)\right] \cdot (2x+1)^{-1} + (3x+2)\frac{d}{dx}(2x+1)^{-1}$$

$$= 3(2x+1)^{-1} + (3x+2)\frac{du^{-1}}{du}\frac{du}{dx}, \quad u = 2x+1$$

$$\frac{du}{dx} = 2$$

$$\frac{df}{dx} = \frac{3}{2x+1} - (3x+2)u^{-2} \cdot 2$$

$$\frac{df}{dx} = \frac{3}{2x+1} - \frac{6x+4}{(2x+1)^2} = \frac{6x+3 - (6x+4)}{(2x+1)^2}$$

$$= \frac{-1}{(2x+1)^2}$$

Ex.4 $f(x) = \left(\frac{3x+2}{2x+1}\right)^5 = u_{(x)}^5$

$$\frac{df}{dx} = \frac{d}{dx}u_{(x)}^5 = \frac{du^5}{du}\frac{du}{dx}$$

$$\frac{df}{dx} = 5u^4\frac{(-1)}{(2x+1)^2} = -5\left(\frac{3x+2}{2x+1}\right)^4\frac{1}{(2x+1)^2}$$

$$= -5\frac{(3x+2)^4}{(2x+1)^6}$$

Ex. 5 $f(x) = 5\sqrt{x^2+3} = 5\sqrt{u} = 5u^{1/2}$

$$\frac{df}{dx} = \frac{d(5u^{1/2})}{du}\frac{du}{dx} = 5\cdot\frac{1}{2}u^{-1/2}\frac{d}{dx}(x^2+3)$$

$$\frac{df}{dx} = \frac{5}{2}\frac{2x}{u^{1/2}} = \frac{5x}{(x^2+3)^{1/2}}$$ //.

Ex. 6 $f(x) = \dfrac{1}{(1-x^2)^{3/2}} = (1-x^2)^{-3/2} = u^{-3/2}$

$$\frac{df}{dx} = \frac{du^{-3/2}}{du}\frac{du}{dx} = -\frac{3}{\cancel{2}}u^{-5/2}(-\cancel{2}x) = \frac{3x}{(1-x^2)^{5/2}}$$ //,

7/10/2020

## DERIVADA DA FUNÇÃO EXPONENCIAL

$$f(x) = a^x \quad , \quad a > 0 \text{ e } a \neq 1.$$

$$\frac{d}{dx} a^x = \lim_{h \to 0} \frac{a^{x+h} - a^x}{h} = a^x \lim_{h \to 0} \frac{a^h - 1}{h}$$

$$\lim_{h \to 0} \frac{a^h - 1}{h} = \ln(a) \quad , \quad \boxed{\ln x = \log_e x}$$

$$\boxed{\frac{d}{dx} a^x = a^x \ln a}$$

CASO $a = e$ $\quad \dfrac{d}{dx} e^x = e^x \cancel{\ln e}^{\,1} \Rightarrow \dfrac{d}{dx} e^x = e^x$ //

## DERIVADA DA FUNÇÃO LOGARÍTMICA

$$f(x) = \log_a x \quad , \quad a > 0 \text{ e } a \neq 1.$$

$$\frac{d}{dx} \log_a x = \lim_{h \to 0} \frac{\log_a(x+h) - \log_a(x)}{h} = \lim_{h \to 0} \frac{\log_a\left(\frac{x+h}{x}\right)}{h}$$

$$u = \frac{h}{x} \Rightarrow \frac{d}{dx} \log_a x = \lim_{u \to 0} \frac{1}{x} \cdot \frac{1}{u} \log_a (1 + u)$$

$h \to 0$, $u \to 0$

$$= \frac{1}{x} \lim_{u \to 0} \log_a (1+u)^{1/u} = \frac{1}{x} \lim_{v \to \infty} \log_a \left(1 + \frac{1}{v}\right)^v$$

$$\boxed{\lim_{v \to +\infty} \left(1 + \frac{1}{v}\right)^v = e} \quad v = \frac{1}{u} \, , \; u \to 0^+ \, , \; v \to +\infty$$

$$\frac{d}{dx}\log_a(x) = \frac{1}{x}\lim_{v\to+\infty}\log_a\left(1+\frac{1}{v}\right)^v$$

$$= \frac{1}{x}\log_a\left[\lim_{v\to\infty}\left(1+\frac{1}{v}\right)^v\right] = \frac{1}{x}\log_a e$$

$$\boxed{\frac{d}{dx}\log_a(x) = \frac{1}{x}\log_a e}$$

CASO $a=e \Rightarrow \frac{d}{dx}\ln(x) = \frac{1}{x}\cancel{\ln e}^{\,=1} \Rightarrow \boxed{\frac{d}{dx}\ln x = \frac{1}{x}}$ //

EXEMPLOS

Ex.1 $f(x) = 2^x$ , $g(x) = \log_2 x$

$\frac{d}{dx}\left(2^x\right) = 2^x \ln 2$ // $\qquad \frac{d}{dx}\log_2 x = \frac{1}{x}\log_2 e$ //

Ex.2 $\frac{d}{dx}e^x = e^x$ , $\frac{d}{dx}\ln x = \frac{1}{x}$

Ex.3 $f(x) = 2^{2x^2+3x-1}$ , $u = 2x^2+3x-1$, $f = 2^u$

$$\frac{d}{dx}f(x) = \frac{d}{du}(2^u)\frac{du}{dx} = (4x+3)\,2^u\ln 2$$

$$\frac{d}{dx}f(x) = (4x+3)\,2^{2x^2+3x-1}\ln 2$$ //

Ex.4 $\quad f(x) = \exp\left(\frac{x+1}{x-1}\right), \quad u = \frac{x+1}{x-1}, \quad f = e^{u}$

$$\frac{d}{dx} f(x) = \frac{d\, e^{u}}{du} \frac{du}{dx} = e^{u} \frac{d}{dx}\left(\frac{x+1}{x-1}\right)$$

$$\frac{d}{dx}\left(\frac{x+1}{x-1}\right) = \frac{1\cdot(x-1) - (x+1)\cdot 1}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

$$\frac{d}{dx} f(x) = \frac{-2}{(x-1)^2} \cdot \exp\left(\frac{x+1}{x-1}\right)$$

Ex. 5 $\quad f(x) = \log_2(3x^2 + 7x - 1), \quad u = 3x^2 + 7 - 1, \quad f = \log_2 u$

$$\frac{d}{dx} f(x) = \frac{d}{du} \log_2 u \; \frac{d(3x^2+7x-1)}{dx} = (6x+7)\, \frac{1}{u} \log_2 e$$

$$\frac{d}{dx} f(x) = \frac{6x+7}{3x^2+7-1} \cdot \log_2 e$$

---

$$\frac{d}{dx} e^{u(x)} = \frac{du}{dx} e^{u}$$

$$\frac{d}{dx} \ln(u(x)) = \frac{du/dx}{u}$$

8/10/2020

## DERIVADA DA FUNÇÃO SENO

$$\frac{d}{dx}\,\text{sen}(x) = \lim_{h \to 0} \frac{\text{sen}(x+h) - \text{sen}(x)}{h}$$

$$\text{sen}(x+h) = \text{sen}(x)\cos(h) + \text{sen}(h)\cos(x)$$

$$\frac{\text{sen}(x+h) - \text{sen}(x)}{h} = \frac{\text{sen}(x)\left(\cos(h) - 1\right)}{h} + \frac{\cos(x)\,\text{sen}(h)}{h}$$

$$\frac{d}{dx}\,\text{sen}(x) = \text{sen}(x) \lim_{h \to 0} \cancelto{0}{\frac{\cos(h) - 1}{h}} + \cos(x) \lim_{h \to 0} \cancelto{1}{\frac{\text{sen}(h)}{h}}$$

$$\boxed{\frac{d}{dx}\,\text{sen}(x) = \cos(x)}$$

## DERIVADA DA FUNÇÃO COSSENO

$$\frac{d}{dx}\cos(x) = \lim_{h \to 0} \frac{\cos(x+h) - \cos(x)}{h}$$

$$\cos(x+h) = \cos(x)\cos(h) - \text{sen}(x)\,\text{sen}(h)$$

$$\frac{\cos(x+h) - \cos(x)}{h} = \cos(x)\frac{\left[\cos(h) - 1\right]}{h} - \frac{\text{sen}(x)\,\text{sen}(h)}{h}$$

$$\frac{d}{dx}\cos(x) = \cos(x) \lim_{h \to 0} \cancelto{0}{\frac{\cos(h) - 1}{h}} - \text{sen}(x) \lim_{h \to 0} \cancelto{1}{\frac{\text{sen}(h)}{h}}$$

$$\boxed{\frac{d}{dx}\cos(x) = -\,\text{sen}(x)}$$

Ex. 1 $f(x) = \mathrm{sen}(x^3 + x^2)$, $u = x^3 + x^2$, $f = \mathrm{sen}(u)$

$$\frac{df}{dx} = \frac{d}{du}\mathrm{sen}(u)\,\frac{du}{dx} = \cos(u)\,(3x^2 + 2x)$$

$$\frac{d}{dx} f(u(x)) = \underbrace{(3x^2+2x)}_{\frac{du'}{dx}} \cdot \underbrace{\cos(x^3+x^2)}_{\cos'(u)} \;//$$

DERIVADA DA FUNÇÃO TANGENTE

$$f(x) = \mathrm{tg}(x) = \frac{\mathrm{sen}(x)}{\cos(x)}$$

$$\frac{d}{dx}\mathrm{tg}(x) = \frac{d}{dx}\left[\frac{\mathrm{sen}(x)}{\cos(x)}\right] = \frac{\left(\frac{d}{dx}\mathrm{sen}x\right)\cos x - \mathrm{sen}x\left(\frac{d}{dx}\cos x\right)}{\cos^2 x}$$

$$\frac{d}{dx}\mathrm{tg}(x) = \frac{\cos^2 x + \mathrm{sen}^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2(x)$$

$$\boxed{\frac{d}{dx}\mathrm{tg}(x) = \frac{1}{\cos^2(x)} = \sec^2(x)}$$

Ex. $f(x) = \mathrm{tg}(\sqrt{x^2+1})$, $u = \sqrt{x^2+1}$, $f = \mathrm{tg}\,u$

$$\frac{d}{dx} f(u(x)) = \frac{d}{du}\mathrm{tg}(u)\,\frac{du}{dx} = \sec^2(u)\,\frac{d}{dx}\sqrt{x^2+1}$$

$$\frac{d}{dx}\underbrace{\sqrt{x^2+1}}_{=v} = \frac{dv^{1/2}}{dv}\,\frac{dv}{dx} = \frac{1}{2}v^{-1/2}\,2x = \frac{x}{\sqrt{x^2+1}}$$

$$= \frac{x\sec^2(\sqrt{x^2+1})}{\sqrt{x^2+1}} \;//$$

# DERIVADA DA SECANTE

$$f(x) = \sec(x) = \frac{1}{\cos(x)}$$

$$\frac{d}{dx}\sec(x) = (\cos(x))^{-1} \quad , \quad u = \cos(x)$$

$$= \frac{d}{dx} u^{-1}_{(x)} = \frac{du^{-1}}{du}\frac{du}{dx} = -1u^{-2}(-\operatorname{sen} x)$$

$$\boxed{\frac{d}{dx}\sec(x) = \frac{\operatorname{sen} x}{\cos^2 x} = \frac{\operatorname{tg} x}{\cos x} = \operatorname{tg} x \cdot \sec x}$$

14/10/2020

# DERIVADA DE COTANGENTE

$$\text{cotg}(x) = \frac{1}{\text{tg}(x)} = \frac{1}{\frac{\text{sen}(x)}{\cos(x)}} = \frac{\cos(x)}{\text{sen}(x)}$$

$$\frac{d}{dx}\text{cotg}(x) = \frac{d}{dx}\left[\frac{\cos(x)}{\text{sen}(x)}\right] = \frac{-\text{sen}(x)\,\text{sen}(x) - \cos(x)\cos(x)}{\text{sen}^2(x)}$$

$$= -\frac{\text{sen}^2(x) + \cos^2(x)}{\text{sen}^2(x)} = -\frac{1}{\text{sen}^2(x)} = -\text{cosec}^2(x)$$

$$\boxed{\frac{d}{dx}\text{cotg}(x) = -\text{cosec}^2(x)}$$

# DERIVADA DA COSSECANTE

$$\text{cosec}(x) = \frac{1}{\text{sen}(x)} = \left(\text{sen}(x)\right)^{-1}$$

$$\frac{d}{dx}\text{cosec}(x) = \frac{d}{dx}\left(\text{sen}(x)\right)^{-1} = \frac{d}{du}u^{-1}\,\frac{du}{dx}, \quad u = \text{sen}\,x$$

$$= -1\,u^{-2}\cos x = -\frac{1}{\text{sen}^2 x}\cos x = -\frac{1}{\text{sen}(x)} \cdot \frac{\cos(x)}{\text{sen}(x)}$$

$$\boxed{\frac{d}{dx}\text{cosec}(x) = -\text{cosec}(x) \cdot \text{cotg}(x)}$$

Ex.1 $f(x) = \text{sen}(x^3+x^2)$

$$\frac{df}{dx} = (3x^2+2x)\cos(x^3+x^2) \quad //$$

Ex.2 $f(x) = \cos(\sqrt{x^3+3})$ , $u = \sqrt{x^3+3}$

$$\frac{df}{dx} = \frac{d\cos(u)}{du}\frac{du}{dx} = -\text{sen}(\sqrt{x^3+3})\frac{d}{dx}\sqrt{x^3+3}$$

$$\frac{d}{dx}\sqrt{x^3+3} = \frac{dv^{1/2}}{dv}\frac{dv}{dx} = \frac{1}{2}v^{-1/2}(3x^2) = \frac{3}{2}\frac{x^2}{\sqrt{x^3+3}}$$

$$\Rightarrow \frac{df}{dx} = -\frac{3}{2}\frac{x^2}{\sqrt{x^3+3}}\,\text{sen}(\sqrt{x^3+3}) \quad //$$

Ex.3 $f(x) = \sec(x)\cdot\text{sen}(x^2)$

$$\frac{df}{dx} = \left(\frac{d}{dx}\sec(x)\right)\text{sen}(x^2) + \sec(x)\frac{d}{dx}\text{sen}(x^2)$$

$$= -(\text{sen}(x))^{-2}\cdot\text{sen}(x^2) + \sec(x)(2x)\cos(x^2)$$

$$\frac{df}{dx} = -\frac{\text{sen}(x^2)}{\text{sen}^2(x)} + 2x\sec(x)\cos(x^2) \quad //$$

$$\frac{d}{dx}f(x) = -\sec^2(x)\,\text{sen}(x^2) + 2x\sec(x)\cos(x^2) \quad //$$

# DERIVADAS DE SENO E COSSENO HIPERBÓLICOS

$$\operatorname{senh}(x) = \frac{e^{x} - e^{-x}}{2}$$

$$\frac{d}{dx}\operatorname{senh}(x) = \frac{1}{2}\frac{d}{dx}\left(e^{x} - e^{-x}\right) = \frac{1}{2}\left(e^{x} - \frac{d}{du}e^{u}\frac{du}{dx}\right),\ u = -x$$

$$\frac{du}{dx} = -1$$

$$\frac{d}{dx}\operatorname{senh}(x) = \frac{1}{2}\left(e^{x} + e^{-x}\right)$$

$$\boxed{\frac{d}{dx}\operatorname{senh}(x) = \cosh(x)}$$

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2}$$

$$\frac{d}{dx}\cosh(x) = \frac{1}{2}\frac{d}{dx}\left(e^{x} + e^{-x}\right) = \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

$$\boxed{\frac{d}{dx}\cosh(x) = \operatorname{senh}(x)}$$

19/10/2020

# Derivada de Função Inversa

Função $u = u(x)$ e sua inversa $v = v(x)$:

$$u(v(x)) = x$$

$$v(u(x)) = x$$

outra notação: $u = f(x)$, $v = f^{-1}(x)$

$$f(f^{-1}(x)) = x \quad , \quad f^{-1}(f(x)) = x .$$

Pela regra da cadeia: $\frac{d}{dx}\left[ u(v(x)) = x \right]$

$$\frac{du}{dv}\frac{dv}{dx} = \frac{dx}{dx} = 1 \Rightarrow \boxed{\frac{dv}{dx} = \frac{1}{\frac{du}{dv}}}$$

Ex. $u = x^2$, $D_u = [0, +\infty)$

$$x = \pm\sqrt{u} = \sqrt{u} \Rightarrow v = \sqrt{x} \ , \ u(v) = v^2 = x.$$

$$\frac{dv}{dx} = 1 \Big/ \frac{du}{dv} = 1 \Big/ \frac{d}{dv}v^2 = 1/2v = 1/2\sqrt{x} \ //$$

$$\frac{dv}{dx} = \frac{d}{dx}\sqrt{x} = \frac{1}{2}\frac{1}{\sqrt{x}} \ //$$

# DERIVADA DO ARCOSSENO

$u = \text{sen}(x)$ , $D_u = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ , $I_u = [-1, 1]$

$v(x) = \text{arcsen}(x) = \text{sen}^{-1}(x)$ , $\frac{dv}{dx}$ ?

$$\frac{dv}{dx} = 1 \Big/ \frac{du}{dv} = 1 \Big/ \frac{d}{dv}\text{sen}(v) = \frac{1}{\cos(v)}$$

$$\cos^2(v) + \text{sen}^2(v) = 1 \Rightarrow \cos(v) = \sqrt{1 - \text{sen}^2(v)}$$

$$u(v(x)) = x \Rightarrow \text{sen}(v) = x \Rightarrow \cos(v) = \sqrt{1 - x^2}$$

$$\boxed{\frac{d}{dx}\text{arcsen}(x) = \frac{1}{\sqrt{1 - x^2}}} \qquad |x| < 1$$

$$\boxed{\frac{d}{dx}\text{arctg}(x) = \frac{1}{1 + x^2}}$$

# DERIVADAS DE ORDEM SUPERIOR

$$f = f(x)$$

1ª derivada: $\dfrac{df}{dx} = f'$

2ª derivada: $\dfrac{d}{dx}\left(\dfrac{df}{dx}\right) = \dfrac{d^2 f}{dx^2} = f''$

Ex. $x(t) = \dfrac{x_0}{2} + v_0(t-t_0) + \dfrac{x_0}{2}\exp\left(-\dfrac{(t-t_0)}{t_0}\right), \quad t \geq t_0$

$$F = ma = m\frac{dv}{dt} = m\frac{d}{dt}\left(\frac{dx}{dt}\right) = m\frac{d^2}{dt^2}x(t)$$

$$\frac{dx}{dt} = v(t) = v_0 + \frac{x_0}{2}\frac{d}{dt}\left(-\left(\frac{t-t_0}{t_0}\right)\right)\exp\left(-\frac{(t-t_0)}{t_0}\right)$$

$$v(t) = v_0 + \frac{x_0}{2}\left(-\frac{1}{t_0}\right)\exp\left[-\frac{(t-t_0)}{t_0}\right] \quad //$$

$$a = \frac{d^2}{dt^2}x(t) = \frac{d}{dt}\left(\frac{dx(t)}{dt}\right) = \frac{d}{dt}v(t) = \frac{x_0}{2}\frac{1}{t_0^2}\exp\left[-\frac{(t-t_0)}{t_0}\right] \quad //$$

$$F = \frac{m}{2}\frac{x_0}{t_0^2}\exp\left[-\frac{(t-t_0)}{t_0}\right]$$

22/10/2020

# Exercícios da lista: Seção 7

18. $f(x) = \ln\left(x^4 + 2x^2\right)$ $\quad u = x^4 + 2x^2$

$$f'(x) = \frac{d}{dx} f = \frac{d}{dx} \ln\left(x^4 + 2x^2\right) = \frac{d \ln u}{du} \frac{du}{dx}$$

$$\Rightarrow f' = \frac{1}{u} \frac{d}{dx}\left(x^4 + 2x^2\right) = \frac{\left(4x^3 + 4x\right)}{x^4 + 2x^2} = \frac{4x^2 + 4}{x^3 + 2x} \;//$$

15. $f(x) = \left(x^2 + 1\right) \exp\left(\frac{x^3 + x^2}{x^2 + 1}\right)$

$$f'(x) = \left(x^2+1\right)' \exp\left(\frac{x^3+x^2}{x^2+1}\right) + \left(x^2+1\right) \frac{d}{dx} \exp\left(\frac{x^3+x^2}{x^2+1}\right)$$

$$= 2x \cdot \exp\left(\frac{x^3+x^2}{x^2+1}\right) + \left(x^2+1\right) \frac{d}{dx}\left(\frac{x^3+x^2}{x^2+1}\right) \exp\left(\frac{x^3+x^2}{x^2+1}\right)$$

$$\frac{d}{dx}\left(\frac{x^3+x^2}{x^2+1}\right) = \frac{\left(x^3+x^2\right)' \cdot \left(x^2+1\right) - \left(x^3+x^2\right) \cdot \left(x^2+1\right)'}{\left(x^2+1\right)^2}$$

$$= \frac{\left(3x^2+2x\right)\left(x^2+1\right) - \left(x^3+x^2\right)(2x)}{\left(x^2+1\right)^2}$$

$$f'(x) = 2x \cdot \exp\left(\frac{x^3+x^2}{x^2+1}\right) + \left[\left(3x^2+2x\right) - \frac{2x^4 + 2x^3}{x^2+1}\right] \exp\left(\frac{x^3+x^2}{x^2+1}\right) \;//$$

23. $f(x) = \cos(\underbrace{\text{sen}(x^2)}_{})$ argumento do cosseno

$$f'(x) = \frac{d}{du}\cos(u)\,\frac{du}{dx}, \quad u = \text{sen}(x^2)$$

$$= -\text{sen}(u)\,\frac{d}{dx}\text{sen}(x^2) = -\text{sen}(u)\,(2x)\cos(x^2)$$

$$f'(x) = -2x\,\text{sen}(\text{sen}(x^2))\cos(x^2)$$

22. $f(x) = \cos\left(\frac{x^4}{4}\right)$ $u = \frac{x^4}{4}$

$$f'(x) = \frac{d}{dx}\cos(u(x)) = \frac{d\cos(u)}{du}\,\frac{du}{dx} = -\text{sen}(u)\,\underbrace{\frac{d}{dx}\left(\frac{x^4}{4}\right)}_{=\frac{4x^3}{4} = x^3}$$

$$f'(x) = -x^3\,\text{sen}\left(\frac{x^4}{4}\right)$$

21. $f(x) = \ln\Big(\underbrace{\frac{1}{x} + \frac{1}{x^2}}_{u}\Big)$

$$f'(x) = \frac{d}{dx}\ln(u(x)) = \frac{d\ln u}{du}\,\frac{du}{dx} = \frac{1}{u}\,\frac{d}{dx}\Big(\underbrace{\frac{1}{x}}_{x^{-1}} + \underbrace{\frac{1}{x^2}}_{x^{-2}}\Big)$$

$$f'(x) = \frac{1}{x^{-1}+x^{-2}}\cdot\left((-1)x^{-2} - 2x^{-3}\right) = -\frac{x^{-2}+x^{-3}}{x^{-1}+x^{-2}} \cdot \frac{x}{x}$$

$$\Rightarrow f'(x) = -\frac{x^{-1}+x^{-2}}{1+x^{-1}}$$

---

$$\frac{d}{dx}e^{\overbrace{\sqrt{x^2+1}}^{u}} = \frac{d}{dx}e^u = \frac{de^u}{du}\cdot\frac{du}{dx} = e^u\,\frac{d}{dx}\overbrace{\sqrt{\underbrace{x^2+1}_{v}}}^{u} = e^u\,\frac{d}{dx}v^{1/2}$$

$$= e^u\,\frac{dv^{1/2}}{dv}\,\frac{dv}{dx} = e^u\,\frac{1}{2}v^{-1/2}\,\frac{d}{dx}(x^2+1) = x\,(x^2+1)^{-1/2}\,e^{\sqrt{x^2+1}}$$

27/10/2020

# Derivação Implícita

Função implícita: $F(x,y) = 0$

Ex.0 $x^2 + y^2 - 4 = 0 \quad , \quad y \geq 0$

$\hookrightarrow$ $y = f(x)$ está definido implicitamente.

$$y^2 = 4 - x^2 \Rightarrow y = \sqrt{4 - x^2}$$

$\hookrightarrow$ $y = f(x)$ definida explicitamente.

Ex.1 $F(x,y) = x^2 + \frac{y}{2} - 1 = 0$

$$\frac{y}{2} = 1 - x^2 \Rightarrow y = 2(1 - x^2) \rightarrow \frac{dy}{dx} = -4x$$

às vezes isso é impossível ou muito difícil.

Pela derivação implícita: $\frac{d}{dx}\left[F(x,y) = 0\right]$

$$\Rightarrow \frac{d}{dx}\left[x^2 + \frac{y}{2} - 1\right] = 0$$

$$\Rightarrow 2x + \frac{y'}{2} = 0 \Rightarrow y' = -4x$$

<u>Ex. 2</u> $F(x,y) = y^2 + x^2 - 4 = 0$

$$\frac{d}{dx}\left[ F(x,y) = 0 \right] \Rightarrow \frac{d}{dx}\left[ y^2 + x^2 - 4 \right] = 0$$

$$\Rightarrow \frac{d}{dx} y^2_{(x)} + 2x = 0 \Rightarrow \frac{d\,y^2}{dy}\frac{dy}{dx} + 2x = 0$$

$$\Rightarrow 2y\,y' + 2x = 0 \Rightarrow y' = \frac{-x}{y_{(x)}} \text{ //}$$

Verificando: $\frac{d}{dx} y = \frac{d}{dx}\sqrt{4 - x^2}$

$$\hookrightarrow \frac{dy}{dx} = \frac{d}{dx} u^{1/2},\ u = 4 - x^2$$

$$\Rightarrow y' = \frac{d}{dx} u^{1/2}_{(x)} = \frac{du^{1/2}}{du}\frac{du}{dx} = \frac{1}{2} u^{-1/2}(-2x)$$

$$\Rightarrow y' = \frac{-x}{\sqrt{4 - x^2}} \text{ /}$$

# DIFERENCIAIS

$$f = f(x)$$

$$\frac{df}{dx} = f' = \lim_{\Delta x \to 0} \frac{\overbrace{f(x+\Delta x) - f(x)}^{\Delta f}}{\underbrace{\Delta x}}$$

Diferencial de $f$ : $\boxed{df = f' \, dx}$

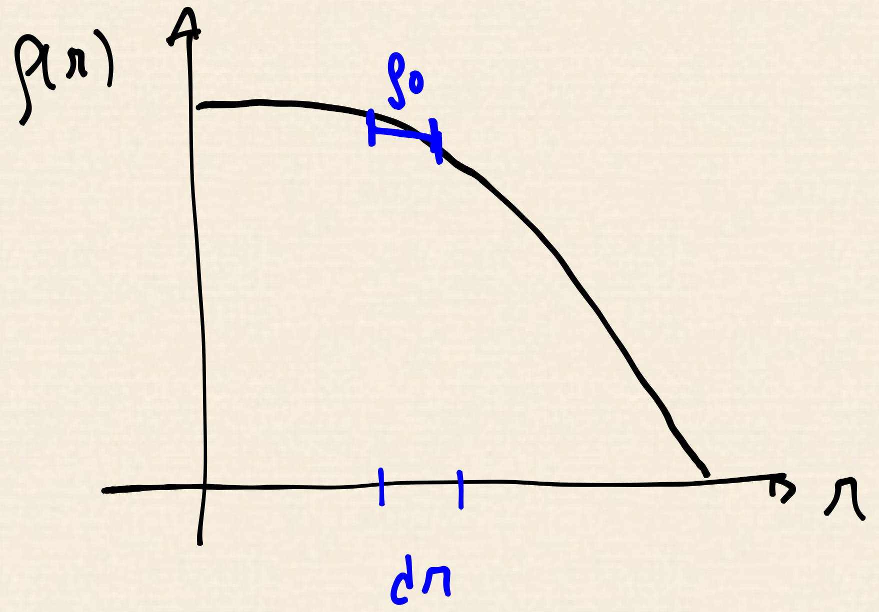
Ex.1 $\quad f = e^{x^2} \;,\; f' = 2x e^{x^2}$

$$\frac{df}{dx} = f' = 2x e^{x^2}$$

$$\Rightarrow \; df = 2x e^{x^2} dx \;//$$

Ex.2 $\quad M = \frac{4}{3}\pi r^3 \rho_0$

$$\frac{dM}{dr} = \frac{4\pi}{\cancel{3}} \cancel{3} r^2 \rho_0 \Rightarrow dM = 4\pi r^2 \rho_0 \, dr$$



Exercícios SEC.7 31

$$f' = \frac{d}{dx} \operatorname{arcsen}\left(\frac{1}{2+x}\right) \qquad \frac{d}{dx} \operatorname{arcsen}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \operatorname{arcsen}(u_{(x)}) = \frac{d}{du} \operatorname{arcsen}(u) \cdot \frac{du}{dx}$$

$$= \frac{1}{\sqrt{1-u^2}} \frac{d}{dx} (2+x)^{-1}$$

$$f' = \frac{1}{\sqrt{1-\left(\frac{1}{2+x}\right)^2}} (-1)(2+x)^{-2} \cdot 1 = -\frac{(2+x)^{-2}}{\sqrt{1-(2+x)^{-2}}} //$$

SEC. 8 2 $f(t) = X_0 \operatorname{sen}(\omega t + a)$, $\frac{d^2 f}{dt^2}$?

$$\frac{df}{dt} = X_0 \cos(\omega t + a) \frac{d}{dt}(\omega t + a)$$

$$\frac{df}{dt} = X_0 \omega \cos(\omega t + a)$$

$$\frac{d^2 f}{dt^2} = \frac{d}{dt}\left(\frac{df}{dt}\right) = -X_0 \omega \operatorname{sen}(\omega t + a) \cdot \frac{d}{dt}(\omega t + a)$$

$$\frac{d^2 f}{dt^2} = -X_0 \omega^2 \operatorname{sen}(\omega t + a) //$$

SEC. 9, 14   $e^y = x + y$

$y = f(x)$?

$\ln\left[ e^y = x + y \right]$

$\ln e^y = \ln(x+y)$

$y = \ln(x+y)$

$\frac{d}{dx}\left[ e^y = x + y \right]$

$\Rightarrow \frac{d}{dx} e^{y(x)} = 1 + y'$

$\frac{d}{dy} e^y \frac{dy}{dx} = 1 + y'$

$y' e^y = 1 + y' \Rightarrow y'(e^y - 1) = 1$

$y' = \frac{1}{e^y - 1}$ //