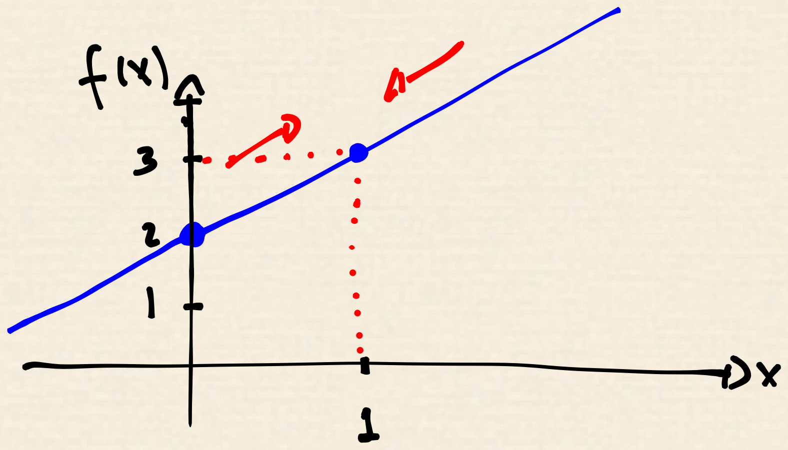
CÁLCULO 1

LIMITES

$$\lim_{x \to c} f(x) = L$$

Ex. 1 $f(x) = x + 2$, $\lim_{x \to 1} f(x)$?

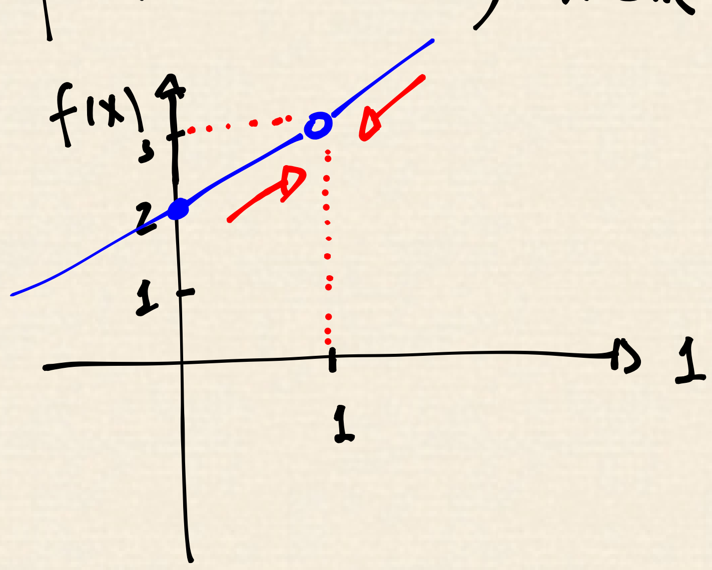


Para qual valor f se aproxima quando $x \to 1$?

$$\lim_{x \to 1} f(x) = 3$$

Ex. 2

$f(x) = x + 2$, $x \in \mathbb{R} \mid x \neq 1$. $\lim_{x \to 1} f(x) = 3$

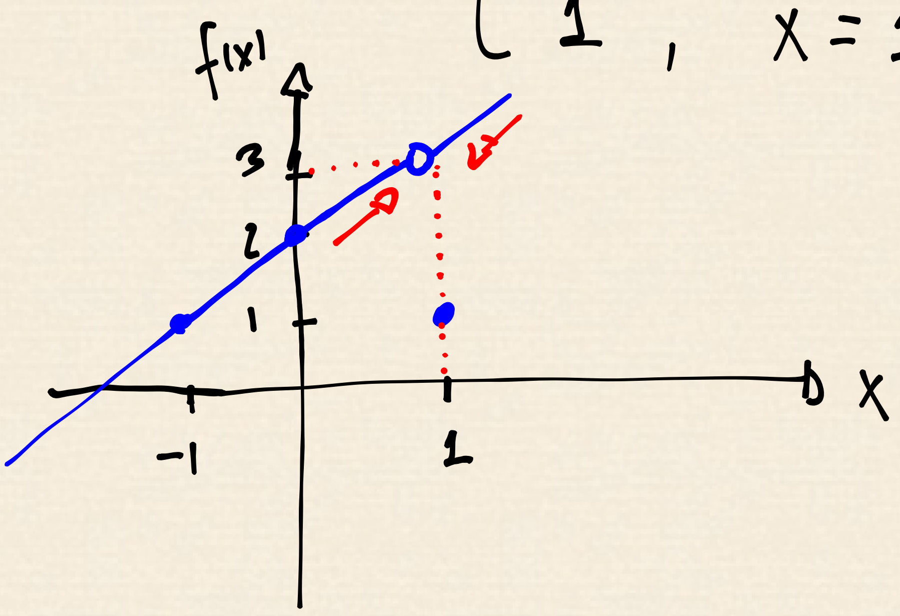


ainda que $f(x=1) \nexists$.

Ex. 3

$$f(x) = \begin{cases} x + 2, & x \in \mathbb{R} \mid x \neq 1 \\ 1, & x = 1 \end{cases}$$

$$\lim_{x \to 1} f(x) = 3$$



ainda que

$$f(x=1) = 1 \neq \lim_{x \to 1} f(x).$$

VIDEO AULA 0

LIMITES

Velocidade instantânea como limite da vel. média

$$v_m = \frac{\Delta X}{\Delta t} = \frac{X(t_2) - X(t_1)}{t_2 - t_1}$$

$$t_2 = t + h \quad , \quad t_1 = t$$

$$v_i = \lim_{h \to 0} v_m = \lim_{h \to 0} \frac{X(t+h) - X(t)}{h}$$

Para $X(t) = x_0 + v_0 t + \frac{1}{2} a t^2$

$$\frac{X(t+h) - X(t)}{h} = \frac{1}{h}\left[\cancel{X_0} + v_0(\cancel{t}+h) + \frac{1}{2}a(t+h)^2 - \cancel{X_0} - \cancel{v_0 t} - \frac{1}{2}at^2\right]$$

$$= \frac{1}{h}\left[v_0 h + \frac{1}{2}a(\cancel{t^2} + 2ht + h^2) - \cancel{\frac{1}{2}at^2}\right]$$

$$= \frac{1}{h}\left[v_0 h + aht + \frac{1}{2}ah^2\right] = v_0 + at + \frac{1}{2}ah$$

$$v_i = \lim_{h \to 0} \frac{X(t+h) - X(t)}{h} = \lim_{h \to 0}\left[v_0 + at + \cancel{\frac{1}{2}ah}^{\,0}\right]$$

$$\boxed{v_i(t) = v_0 + at}$$

$$v_i(t) = \frac{dX(t)}{dt}$$

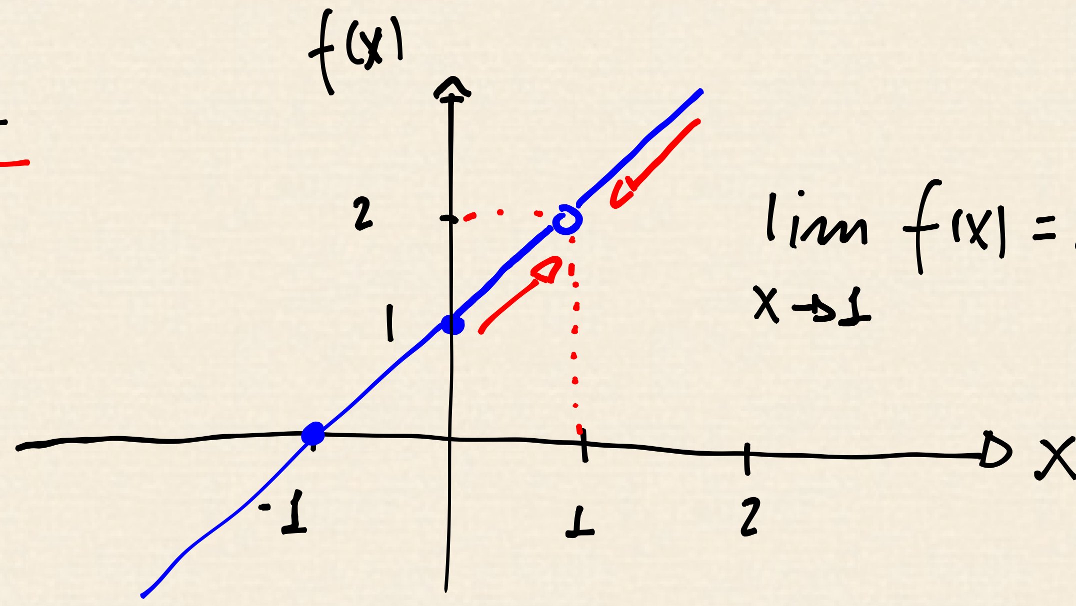
Derivada em relação ao tempo de $X(t)$.

$\underline{\text{Ex.}}$ $\lim\limits_{x \to 1} \dfrac{x^2-1}{x-1}$?

Análise da função: $f(x) = \dfrac{x^2-1}{x-1}$, $D_f = (-\infty, 1) \cup (1, +\infty)$

f tem forma alternativa: $\dfrac{x^2-1}{x-1} = \dfrac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = x+1$

$f = x+1$, $\underline{x \neq 1}$



$\lim\limits_{x \to 1} f(x) = 2$

$\lim\limits_{x \to 1} \dfrac{x^2-1}{x-1} = \text{“}\dfrac{0}{0}\text{”}$ $\underline{\text{INDETERMINAÇÃO}}$

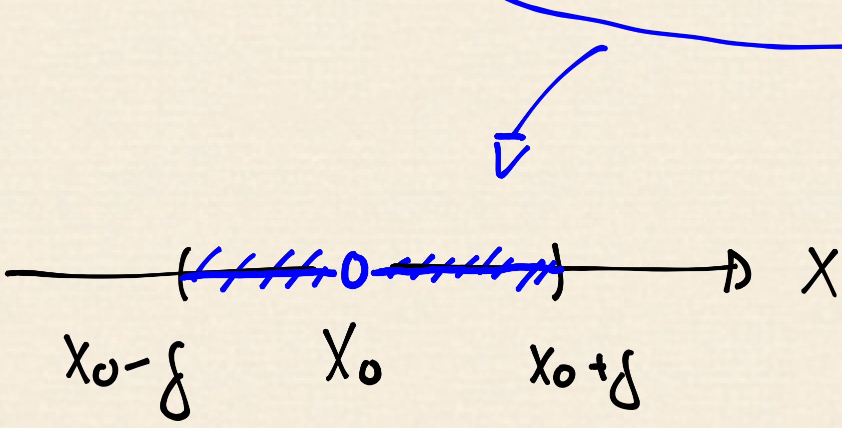
$\lim\limits_{x \to 1} \dfrac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = \lim\limits_{x \to 1} (x+1) = 2$ //

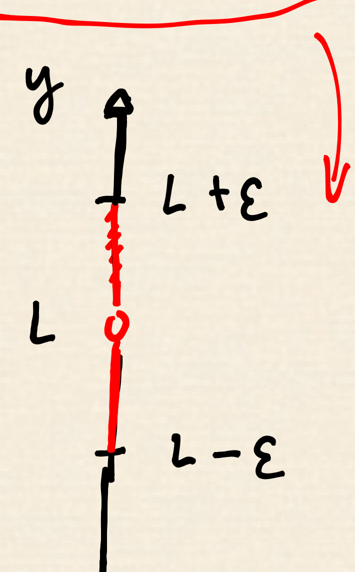
DEFINIÇÃO FORMAL DO LIMITE

$$\lim_{x \to x_0} f(x) = L$$

se para qualquer $\varepsilon > 0$ existir um $\delta > 0$ tal que

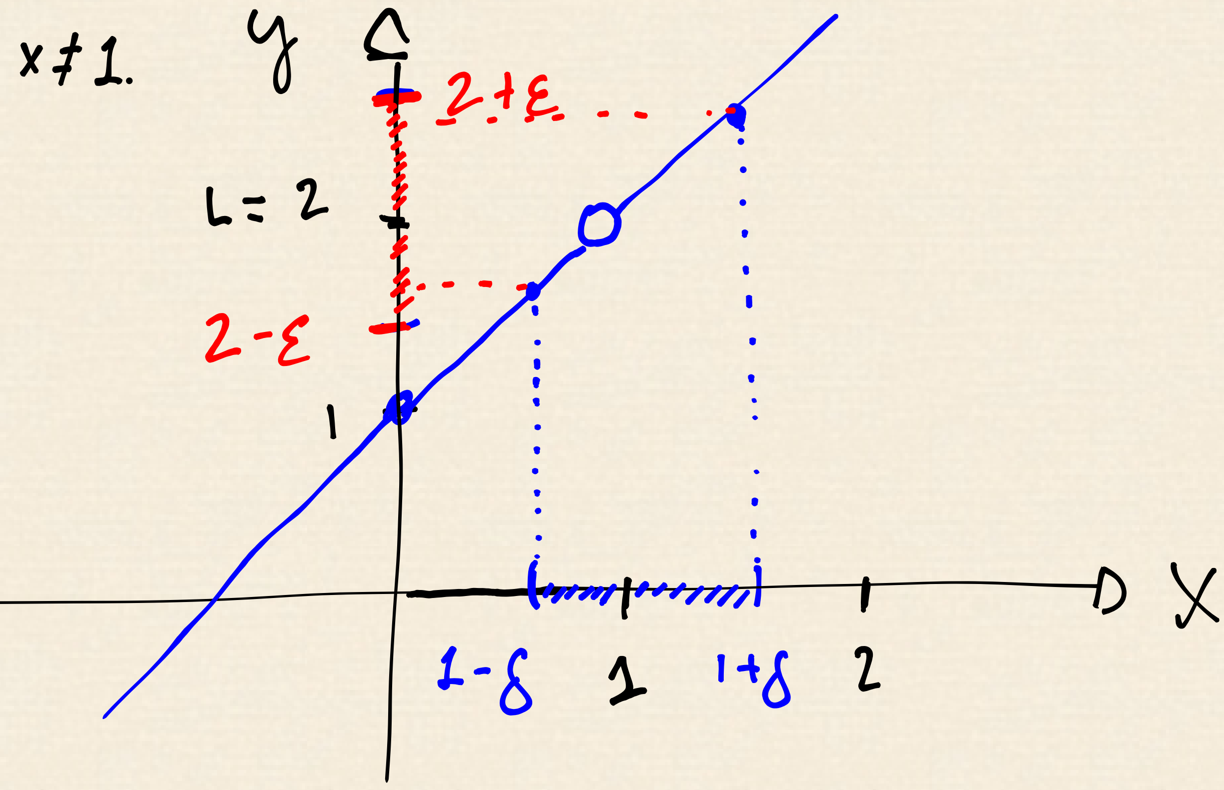
$$\underbrace{0 < |x - x_0| < \delta}_{} \Rightarrow \underbrace{|f(x) - L| < \varepsilon}_{}$$





$f(x) = x+1, \ x \neq 1.$

$$\lim_{x \to 1} f(x) = 2$$



$$\lim_{x \to 1} f(x) = 2 \quad , \qquad |x - x_0| < \delta$$

$$|x - 1| < \delta$$

Função Módulo : $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$$|x-1| < \delta \Rightarrow \begin{cases} x - 1 < \delta \\ -(x-1) < \delta \rightarrow x - 1 > -\delta \end{cases} \Rightarrow -\delta < x - 1 < \delta$$

$$-\delta < x - 1 < \delta \Rightarrow \boxed{1 - \delta < x < 1 + \delta} \quad A$$

$$|f(x) - L| < \varepsilon \Rightarrow |x + 1 - 2| < \varepsilon$$

$$\Rightarrow -\varepsilon < x - 1 < \varepsilon$$

$$\Rightarrow \boxed{1 - \varepsilon < x < 1 + \varepsilon} \quad B$$

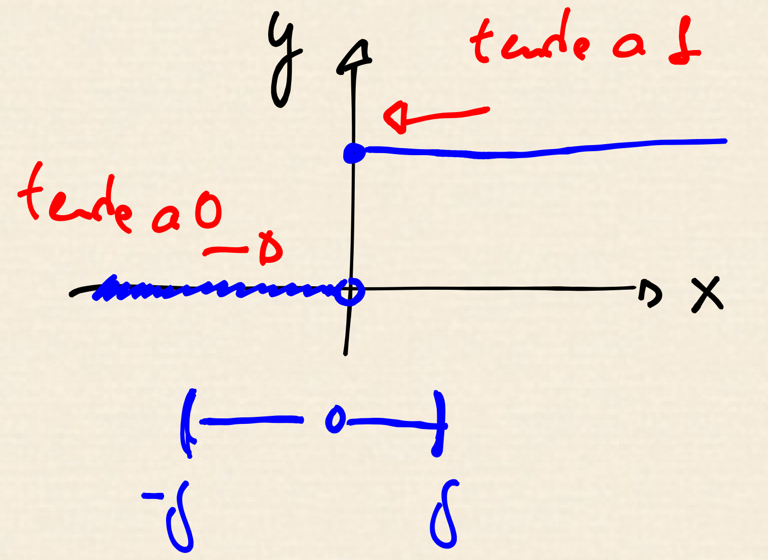
Para qualquer ε o intervalo $A \subset B$?

contido

Sim, basta escolher $\delta = \varepsilon$.

Ex. 1 Limite que não existe.

$$f(x) = \begin{cases} 0 & , x < 0 \\ 1 & , x \geq 0 \end{cases}$$



$\lim\limits_{x \to 0} f(x) = 1$?

$|x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$ $\quad$ $f(x)$ p/ $x > 0$

$-\delta < x < \delta$ $\quad$ Para $x > 0 \Rightarrow |1 - 1| < \varepsilon$

$0 < \varepsilon$ ✓

Para $x < 0 \Rightarrow |0 - 1| < \varepsilon$

$1 < \varepsilon$

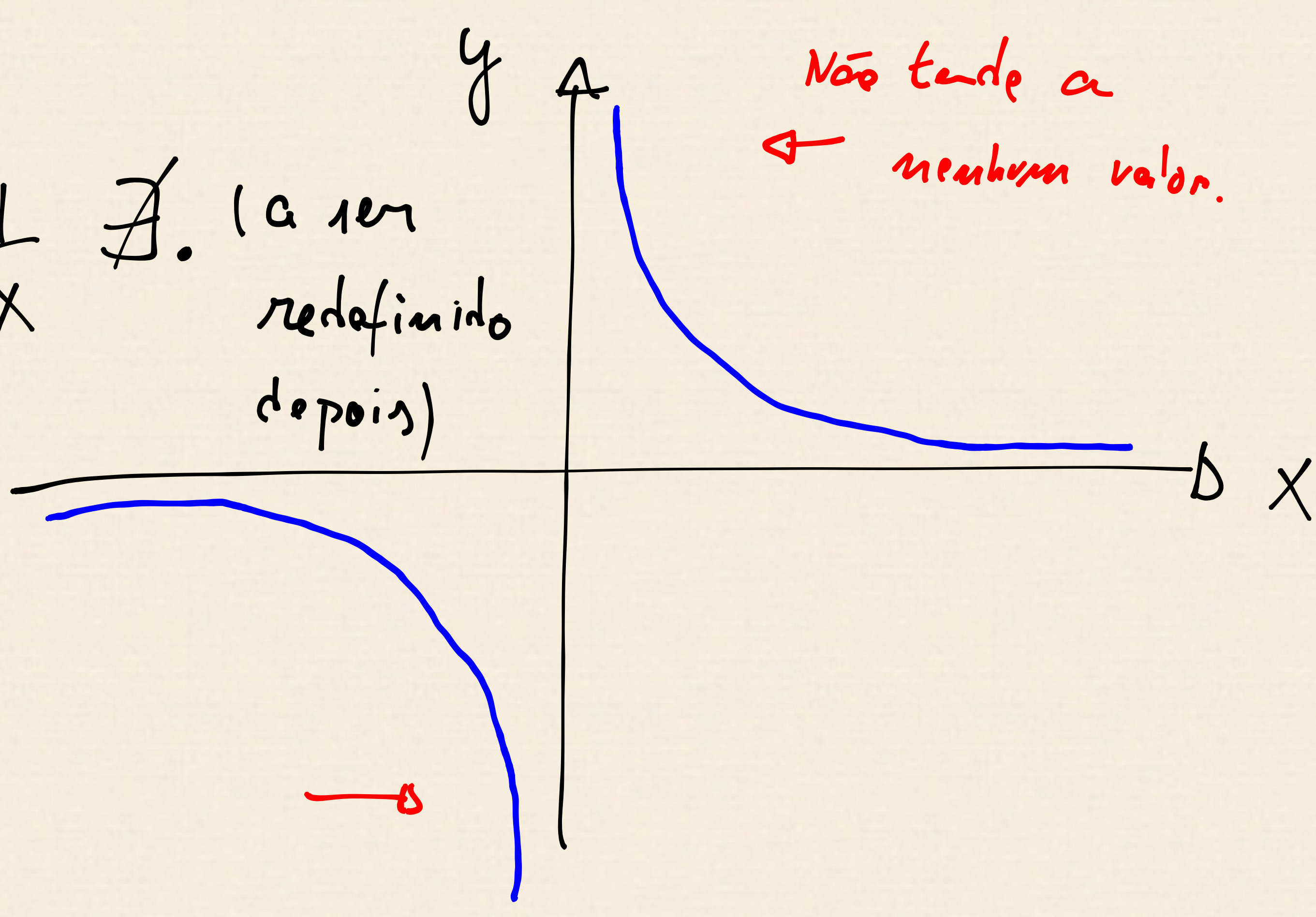
Devemos ter ε qualquer,

mas achamos $\varepsilon > 1$. Portanto $\lim\limits_{x \to 0} f(x) \nexists$.

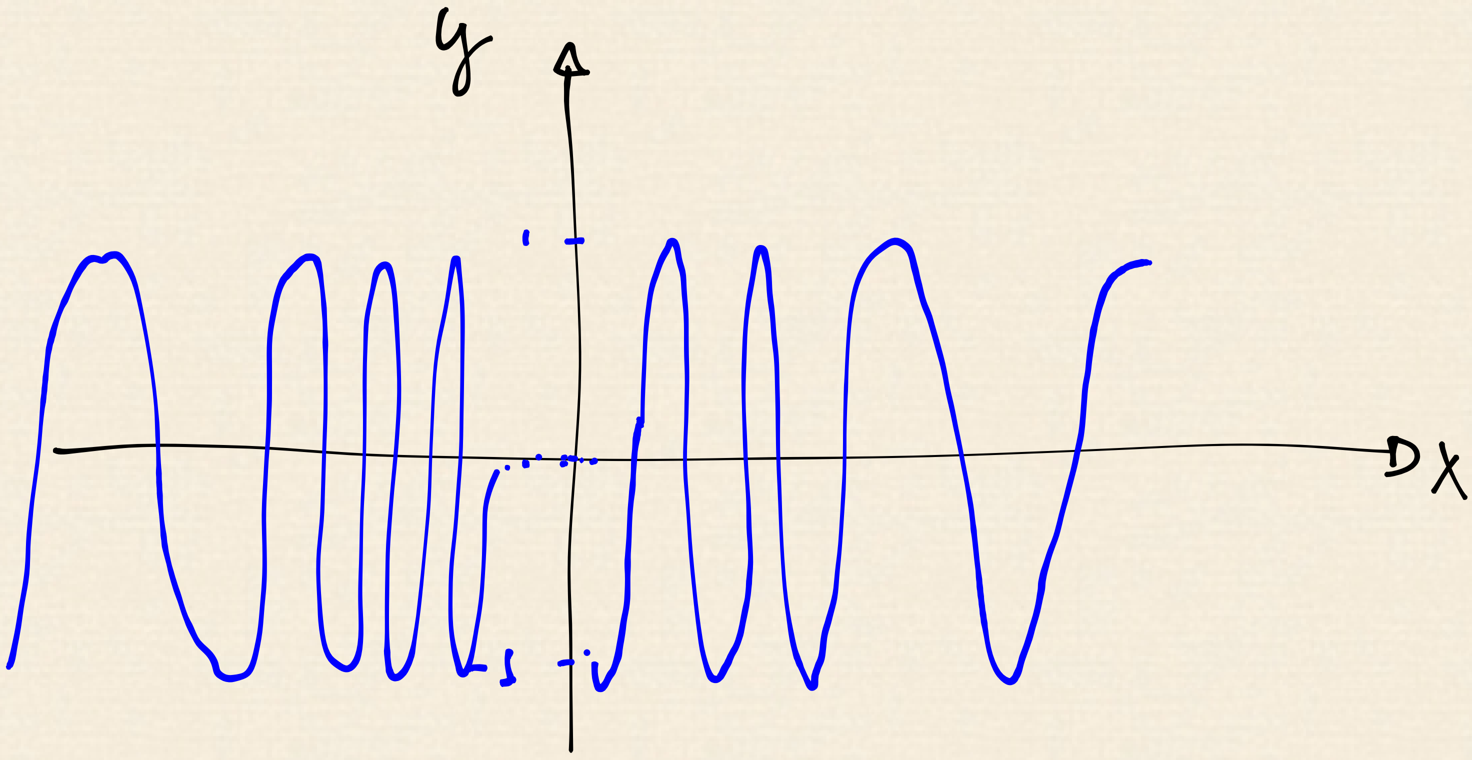
OBS: o mesmo ocorre para $\lim\limits_{x \to 0} f(x) = 0$.

Ex. 2 $\quad f(x) = \dfrac{1}{x}$, $\lim\limits_{x \to 0} f(x)$?

$\lim\limits_{x \to 0} \dfrac{1}{x} \nexists$. (a ser redefinido depois)



Ex. 3 $h(x) = \text{sen}(1/x)$, $\lim_{x \to 0} h(x)$? $\to \nexists$



Limites de funções racionais

$$f(x) = \frac{p(x)}{q(x)}$$, p e q são funções polinomiais.

Pontos sensíveis para o cálculo de limites

$$q(x) = 0.$$

Ex. $f(x) = \dfrac{x^2+3x-4}{x^2-3x+2}$, $\lim_{x \to 1} f(x)$? e $\lim_{x \to 2} f(x)$?

$$x^2 - 3x + 2 = 0$$

$$x = \frac{1}{2}\left[3 \pm \sqrt{9-8}\right]$$

$x_1 = 2$ e $x_2 = 1$

$$x^2 - 3x + 2 = (x-2)(x-1)$$

$\lim_{x \to 1} f(x)$?

Vamos verificar se $p(x)$ é fatorável por $(x-1)$.

$$\begin{array}{rr|l}
x^2 + 3x - 4 & & x - 1 \\ \cline{3-3}
-x^2 + x & & x + 4 \\ \cline{1-1}
4x - 4 & & \\
-4x + 4 & & \\ \cline{1-1}
0 & &
\end{array}$$

Resultado da divisão: $x^2+3x-4 = (x-1)(x+4)$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{\cancel{(x-1)}(x+4)}{\cancel{(x-1)}(x-2)} = \lim_{x \to 1} \frac{x+4}{x-2} = -5 \;//$$

$\lim_{x \to 2} f(x)$?

$$\begin{array}{rl|l} x^2 & +3x-4 & x-2 \\ \cline{3-3} -x^2 & +2x & x+5 \\ \cline{1-2} & 5x-4 & \\ & -5x+10 & \\ \cline{2-2} & 6 & \end{array}$$

$$\Rightarrow x^2+3x-4 = (x-2)(x+5)+6$$

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{(x-2)(x+5)+6}{(x-2)(x-1)} = \lim_{x \to 2} \left[\frac{\cancel{(x-2)}(x+5)}{\cancel{(x-2)}(x-1)} + \frac{6}{(x-2)(x+5)} \right]$$

Ainda tende a zero.

$$\lim_{x \to 2} f(x) \nexists.$$

Seja $f(x)$ e $g(x)$ funções polinomiais:

$$\lim_{x \to c} f(x) = f(c).$$

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{f(c)}{g(c)} \quad \text{se } g(c) \neq 0.$$

VIDEO AULA 1 - LIMITES

Ex. $f(x) = \dfrac{x^2+3x-4}{x^2-3x+2}$, $\lim\limits_{x\to 1} f(x)$? , $\lim\limits_{x\to 2} f(x)$?

$f(x) = \dfrac{p(x)}{q(x)}$. $q(1) = 0$ e $q(2) = 0$.

$x \to 1$

$$
\begin{array}{r|l}
x^2 - 3x + 2 & x-1 \\
\hline
-x^2 + x & x-2 \\
-2x + 2 & \\
+2x - 2 & \\
0 &
\end{array}
\Rightarrow x^2-3x+2 = (x-1)(x-2)
$$

$$
\begin{array}{r|l}
x^2 + 3x - 4 & x-1 \\
\hline
-x^2 + x & x+4 \\
4x - 4 & \\
-4x + 4 & \\
0 &
\end{array}
\Rightarrow x^2+3x-4 = (x-1)(x+4)
$$

$$f(x) = \frac{\cancel{(x-1)}(x+4)}{\cancel{(x-1)}(x-2)} = \frac{x+4}{x-2}, \quad \lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x+4}{x-2} = -5 \;//$$

$x \to 2$

$$
\begin{array}{r|l}
x^2 + 3x - 4 & x-2 \\
\hline
-x^2 + 2x & x+5 \\
5x - 4 & \\
-5x + 10 & \\
6 &
\end{array}
\Rightarrow x^2+3x-4 = (x-2)(x+5)+6
$$

$$= x^2 - 2x + 5x - 10 + 6$$

$$= x^2 + 3x - 4$$

$$f(x) = \frac{(x-2)(x+5)+6}{(x-2)(x-1)} = \frac{\cancel{(x-2)}(x+5)}{\cancel{(x-2)}(x-1)} + \frac{6}{(x-2)(x-1)} = \frac{x+5}{x-1} + \frac{6}{(x-2)(x-1)}$$

$$\lim_{x\to 2} f(x) \nexists. \;//$$

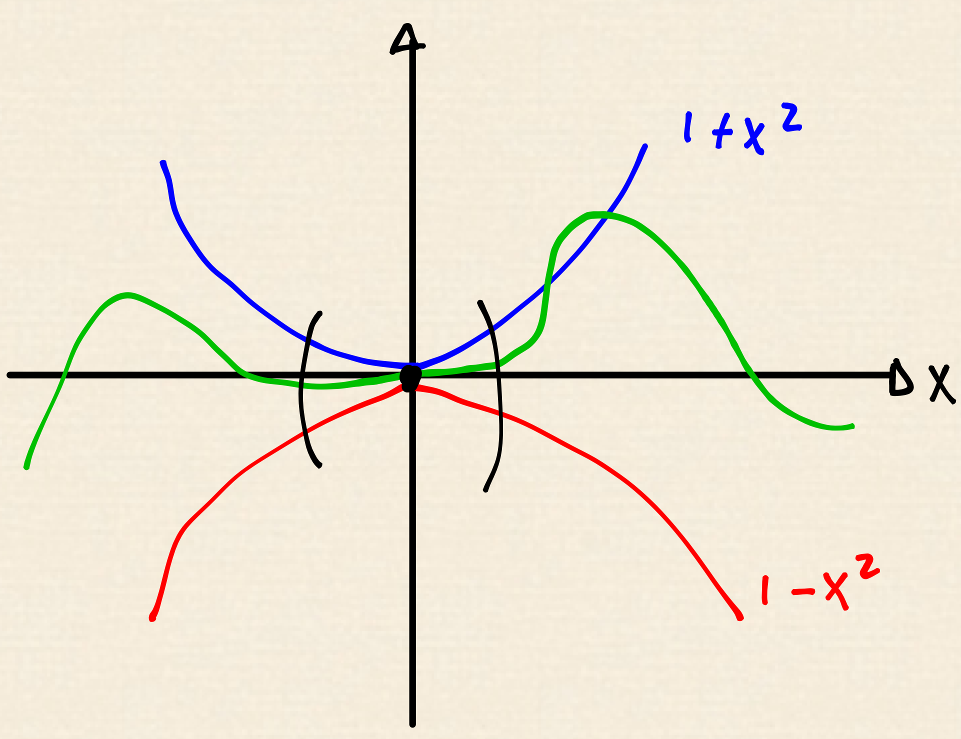
Ex. $\lim\limits_{x \to 0} \dfrac{\sqrt{x^2+100}-10}{x^2} = "\dfrac{0}{0}"$ Indeterminação.

$$\frac{\left(\sqrt{x^2+100}-10\right)}{x^2} \cdot \frac{\left(\sqrt{x^2+100}+10\right)}{\left(\sqrt{x^2+100}+10\right)} = \frac{x^2+\cancel{100} \cancel{-10\left(\sqrt{x^2+100}\right)} \cancel{+10\left(\sqrt{x^2+100}\right)} \cancel{-100}}{x^2\left(\sqrt{x^2+100}+10\right)}$$

$$= \frac{\cancel{x^2}}{\cancel{x^2}\sqrt{x^2+100}+10} = \frac{1}{\sqrt{x^2+100}+10}$$

$$\lim_{x \to 0} \frac{\sqrt{x^2+100}-10}{x^2} = \lim_{x \to 0} \frac{1}{\sqrt{x^2+100}+10} = \frac{1}{20} //$$

TEOREMA DO CONFRONTO



$$1 - x^2 \leq u(x) \leq 1 + x^2$$

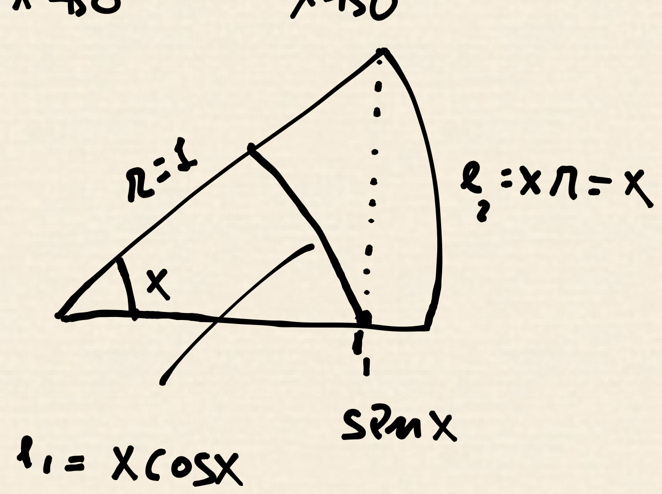
$$\lim_{x \to 0} (1-x^2) = \lim_{x \to 0} (1+x^2) = 0$$

$$\Rightarrow \lim_{x \to 0} u(x) = 0$$

Limite Fundamental: $\lim\limits_{x \to 0} \dfrac{\operatorname{sen}(x)}{x}$

$1 > \dfrac{\operatorname{sen}(x)}{x} > \cos(x)$ no entorno de $x = 0$.

$$\lim_{x \to 0} 1 = \lim_{x \to 0} \cos x = 1 \Rightarrow \boxed{\lim_{x \to 0} \frac{\operatorname{sen}(x)}{x} = 1}$$



Ex.1 $L = \lim_{x\to 0} \dfrac{sen(6x)}{3x} = \lim_{x\to 0} 2\cdot\dfrac{sen(6x)}{2\cdot 3x} = 2\lim_{6x\to 0} \dfrac{sen(6x)}{6x}$

$u(x) = 6x$

$\lim_{x\to 0} u(x) = 0 \Rightarrow L = 2 \lim_{u\to 0} \dfrac{sen\,u}{u} = 2$ //

Ex.2 $\lim_{x\to 0} \dfrac{sen(2x)}{sen(5x)} = \lim_{x\to 0} \dfrac{2x[sen(2x)/2x]}{5x[sen(5x)/5x]}$

$$\boxed{\begin{aligned} u &= 2x \\ v &= 5x \end{aligned}}$$

$= \dfrac{2}{5} \lim_{x\to 0} \dfrac{\dfrac{sen(2x)}{2x}}{\dfrac{sen(5x)}{5x}} = \dfrac{2}{5} \dfrac{\lim_{u\to 0} sen(u)/u}{\lim_{v\to 0} sen(v)/v} = \dfrac{2}{5}$ //

VIDEO AULA 2

LIMITES LATERAIS

NOTAÇÃO: $\lim_{x \to x_0} f(x) = L$ – limite bilateral.

$\lim_{x \to x_0^+} f(x) = L_D$ – limite lateral à direita.

$\lim_{x \to x_0^-} f(x) = L_E$ – limite lateral à esquerda.

LIMITE LATERAL À DIREITA

$\lim_{x \to x_0^+} f(x) = L_D$ existe se para $\forall\, \varepsilon > 0$

existir um $\delta > 0$ tal que

$$x_0 < x < x_0 + \delta \Rightarrow |f(x) - L_D| < \varepsilon$$

LIMITE LATERAL À ESQUERDA

$\lim_{x \to x_0^-} f(x) = L_E$ existe se para $\forall\, \varepsilon > 0$

existir um $\delta > 0$ tal que

$$x_0 - \delta < x < x_0 \Rightarrow |f(x) - L_E| < \varepsilon$$

TEOREMA DA EXISTÊNCIA DO LIMITE

Seja f uma função real; x_0, a e $b \in \mathbb{R}$ tais que (a, x_0) e $(x_0, b) \subset D_f$. Então

$$\boxed{\lim_{x \to x_0^+} f(x) = \lim_{x \to x_0^-} f(x) = L \Leftrightarrow \lim_{x \to x_0} f(x) = L}$$

Ex. $f(x)=\sqrt{4-x^2}$

$x^2+y^2=a^2$ círculo

$y=\pm\sqrt{a^2-x^2}$

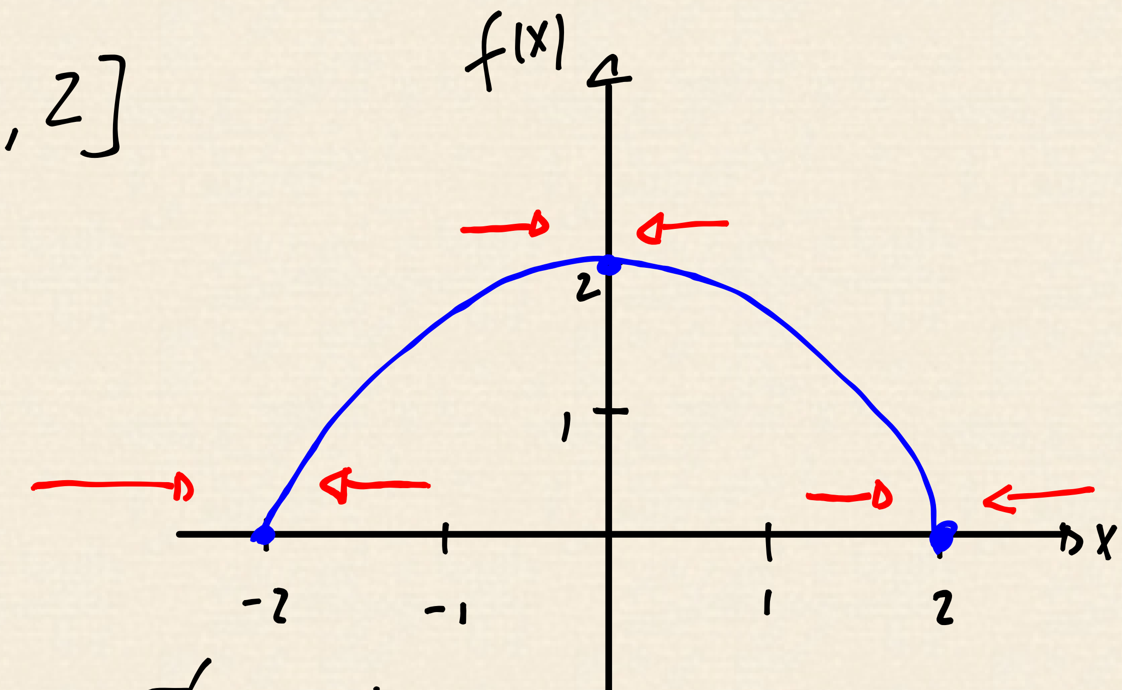
a - raio

$\lim_{x\to -2^+} f(x)$, $\lim_{x\to -2^-} f(x)$

$\lim_{x\to 0^-} f(x)$, $\lim_{x\to 0^+} f(x)$

$\lim_{x\to 2^-} f(x)$, $\lim_{x\to 2^+} f(x)$?

$D_f=[-2,2]$



$\lim_{x\to -2^-} f(x)\ \nexists$, $\lim_{x\to -2^+} f(x)=0 \Rightarrow \lim_{x\to -2} f(x)\ \nexists$

$\lim_{x\to 0^-} f(x)=2$, $\lim_{x\to 0^+} f(x)=2 \Rightarrow \lim_{x\to 0} f(x)=2$.

$\lim_{x\to 2^-} f(x)=0$, $\lim_{x\to 2^+} f(x)\ \nexists \Rightarrow \lim_{x\to 2} f(x)\ \nexists$.

VIDEO AULA 2b

LIMITES NO INFINITO

DEF. 1 O limite $\lim_{x \to \infty} f(x) = L$ existe

se $\forall\, \varepsilon > 0\ \exists\ M > 0$ tal que

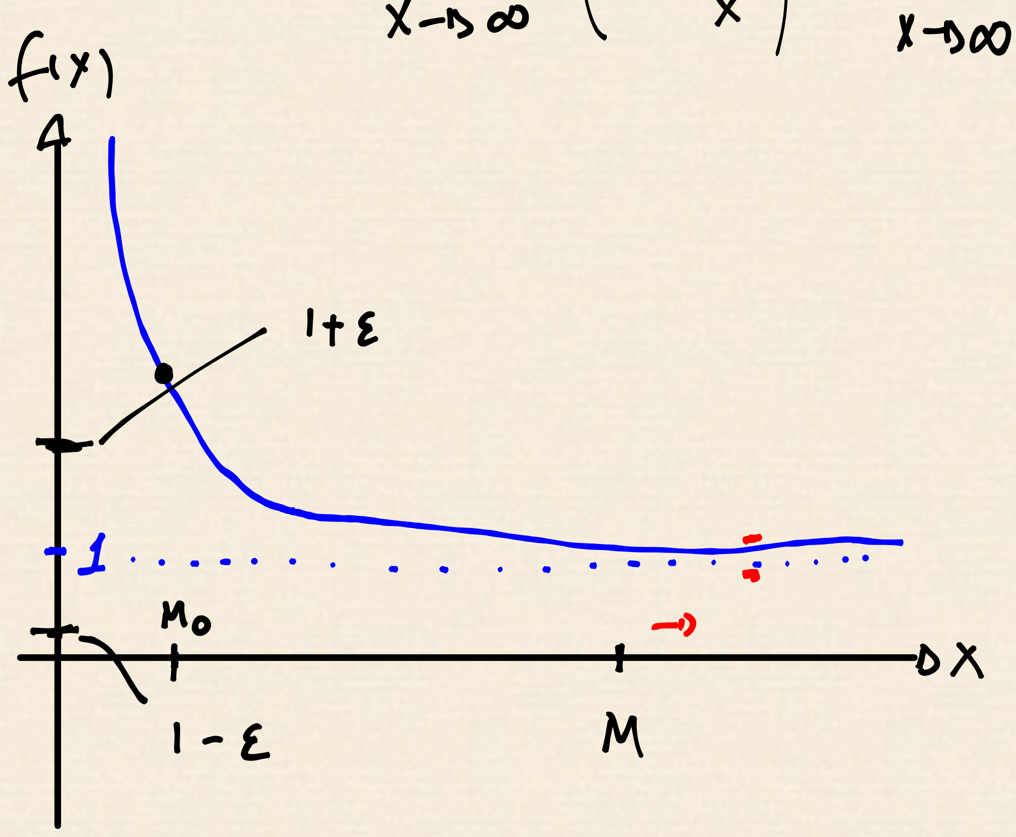
$$x > M \Rightarrow |f(x) - L| < \varepsilon .$$

DEF. 2 O limite $\lim_{x \to -\infty} f(x) = L$ existe

se $\forall\, \varepsilon > 0\ \exists\ N > 0$ tal que

$$x < -N \Rightarrow |f(x) - L| < \varepsilon .$$

Ex. 1 $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right) = \lim_{x\to\infty} 1 + \lim_{x\to\infty} \cancel{\frac{1}{x}}^{\to 0} = 1 \,//$



$|f(x) - L| < \varepsilon$

$-\varepsilon < f(x) - 1 < \varepsilon$

$1 - \varepsilon < f(x) < \varepsilon + 1$

Pela definição:

$\left|\frac{1}{x} - 0\right| < \varepsilon \to \quad \frac{1}{x} < \varepsilon \Leftarrow x > M\ ?$

e.g. $\varepsilon = 0{,}1 \to x > 10,\ M > 10$ ok

$\varepsilon = 0{,}01 \to x > 100,\ M > 100$ ok

$\forall\, \varepsilon,\ M > 1/\varepsilon .$ ok

Ex. 2 $\displaystyle\lim_{x\to\infty} \frac{-2x^2+x+2}{x^2+2x-1} = \text{"}\frac{-\infty}{+\infty}\text{"}$ INDETERMINAÇÃO

$$\lim_{x\to\infty} \frac{\cancel{x^2}}{\cancel{x^2}} \cdot \frac{(-2+\underbrace{1/x}_{\to 0}+\underbrace{2/x^2}_{\to 0})}{(1+\underbrace{2/x}_{\to 0}-\underbrace{1/x^2}_{\to 0})} = -2 \,//$$

Ex. 3 $\displaystyle\lim_{x\to-\infty} \frac{x^2-4x+2}{2x^3-1} = \text{"}\frac{+\infty}{-\infty}\text{"}$ INDETERMINAÇÃO

$$L = \lim_{x\to-\infty} \frac{x^2}{x^3} \cdot \frac{(1-4/x+2/x^2)}{(2-1/x^3)} = \lim_{x\to-\infty} \left(\underbrace{\frac{1}{x}}_{\to 0}\right) \cdot \frac{(1-\underbrace{4/x}_{\to 0}+\underbrace{2/x^2}_{\to 0})}{(2-\underbrace{1/x^3}_{\to 0})}$$

$L = 0 \,//$

Ex. 4 $$\lim_{h\to-\infty} \frac{2-h+\text{sen}(h)}{h+\cos(h)} = \lim_{h\to-\infty} \frac{\cancel{h}}{\cancel{h}} \cdot \frac{(\underbrace{2/h}_{\to 0}-1+\underbrace{\text{sen}(h)/h}_{\to 0})}{(1+\underbrace{\cos(h)/h}_{\to 0})} = -1 \,//$$

$$\lim_{h\to\pm\infty} \frac{\text{sen}(h)}{h} \sim \lim_{h\to\pm\infty} \frac{(\pm 1)}{h} = 0$$

seno ou cosseno são funções limitadas.

- - - - - - - - - - - - -

$h(x) = f(x)\, g(x)$, $\displaystyle\lim_{x\to a} f(x) = 0$, $|g(x)| \leq N$

$\displaystyle\lim_{x\to a} h(x) = ?$, $|h| = |f\, g| = |f|\,|g| \leq |f|\, N$

$|h| \leq |f| N \to -|f| N \leq h \leq |f| N$

$\displaystyle\lim_{x\to a} \pm |f| N = 0 \Rightarrow \lim_{x\to a} h = 0$

Exercícios da Lista, Ser. 1

① $L = 2t$

② $L = 3t^2$

⑤ $\lim\limits_{x \to 3} \dfrac{x^3 - 2x^2 - 2x - 3}{2x^2 - 4x - 6} = \text{"}\dfrac{0}{0}\text{"}$

$$\begin{array}{rrr|l} 2x^2 & -4x & -6 & x-3 \\ \cline{4-4} -2x^2 & +6x & & 2x+2 \\ \cline{1-2} & 2x & -6 & \\ & -2x & +6 & \\ \cline{2-3} & & 0 & \end{array} \Rightarrow 2x^2 - 4x - 6 = (x-3)(2x+2)$$

$$\begin{array}{rrrr|l} x^3 & -2x^2 & -2x & -3 & x-3 \\ \cline{5-5} -x^3 & +3x^2 & & & x^2+x+1 \\ \cline{1-2} & x^2 & -2x & -3 & \\ & -x^2 & +3x & & \\ \cline{2-3} & & x & -3 & \\ & & -x & +3 & \\ \cline{3-4} & & & 0 & \end{array} \Rightarrow x^3 - 2x^2 - 2x - 3 = (x-3)(x^2+x+1)$$

$$L = \lim_{x \to 3} \frac{\cancel{(x-3)}(x^2+x+1)}{\cancel{(x-3)}(2x+2)} = \lim_{x \to 3} \frac{x^2+x+1}{2x+2} = \frac{13}{8} //$$

⑨ $L = \lim\limits_{x \to 1} \dfrac{x^{3/2} - 1}{\sqrt{x} - 1} = \text{"}\dfrac{0}{0}\text{"}$, $\dfrac{x^{3/2} - 1}{x^{1/2} - 1} \cdot \dfrac{(x^{1/2}+1)}{(x^{1/2}+1)} = \dfrac{x^2 - x^{1/2} + x^{3/2} - 1}{x - 1}$

não ajudou.

$x = u^2$, $\sqrt{x} = \sqrt{u^2} = +u$

$x^{3/2} = \sqrt{x^3} = \sqrt{(u^2)^3} = \sqrt{u^6} = (u^6)^{1/2} = u^3$

$\lim\limits_{x \to 1} u = \lim\limits_{x \to 1} \sqrt{x} = 1$

$x \to 1$, $u \to 1$

$L = \lim\limits_{u \to 1} \dfrac{u^3 - 1}{u - 1} \ldots$

(14) $\lim\limits_{x \to 1} \dfrac{\text{sen}(x-1)}{\text{sen}(-x+1)}$, $\text{sen}(-a) = -\text{sen}(a)$

Função Ímpar

$$= \lim_{x \to 1} \frac{\text{sen}(x-1)}{\text{sen}[-(x-1)]} = \lim_{x \to 1} \frac{\cancel{\text{sen}(x-1)}}{-\cancel{\text{sen}(x-1)}} = -1 \;//$$

(17) $\lim\limits_{x \to 0} \dfrac{(1-\cos x)\,\text{sen}(1-\cos(x))}{1 - 2\cos x + \cos^2 x}$, $u = 1 - \cos x$

$u^2 = 1 - 2\cos x + \cos^2 x$

$\lim\limits_{x \to 0} u = 0$

$x \to 0$, $u \to 0$

$$= \lim_{u \to 0} \frac{\cancel{u}\,\text{sen}(u)}{u^{\cancel{2}}}$$

$$= \lim_{u \to 0} \frac{\text{sen}(u)}{u} = 1 \;//$$

(3) $f(x) = \dfrac{x^2 + x - 2}{x^2 - x}$, $\lim\limits_{x \to 1} f(x)$? , $\lim\limits_{x \to -1} f(x)$

$$\lim_{x \to 1} \frac{x^2 + x - 2}{x(x-1)} = \lim_{x \to 1} \frac{\cancel{(x-1)}(x+2)}{x\,\cancel{(x-1)}} = 3 \;//$$

$$\begin{array}{l|l} x^2 + x - 2 & x - 1 \\ \hline -x^2 + x & x + 2 \\ \quad 2x - 2 & \\ \quad -2x + 2 & \\ \quad\quad 0 & \end{array}$$

$$\lim_{x \to -1} \frac{x^2 + x - 2}{x(x-1)} = \frac{-2}{2} = -1 \;//$$

continuação do (9)

$$\begin{array}{l|l} u^3 - 1 & u - 1 \\ \hline -u^3 + u^2 & u^2 + u + 1 \\ \quad u^2 - 1 & \\ \quad -u^2 + u & \\ \quad\quad u - 1 & \\ \quad\quad -u + 1 & \\ \quad\quad\quad 0 & \end{array}$$

$$\lim_{u \to 1} \frac{\cancel{(u-1)}(u^2+u+1)}{\cancel{(u-1)}} = 3 \;//$$

LISTA 1, SEC. 1

10. $\lim\limits_{x \to -2} \dfrac{x+2}{\sqrt{x^2+5}-3}$

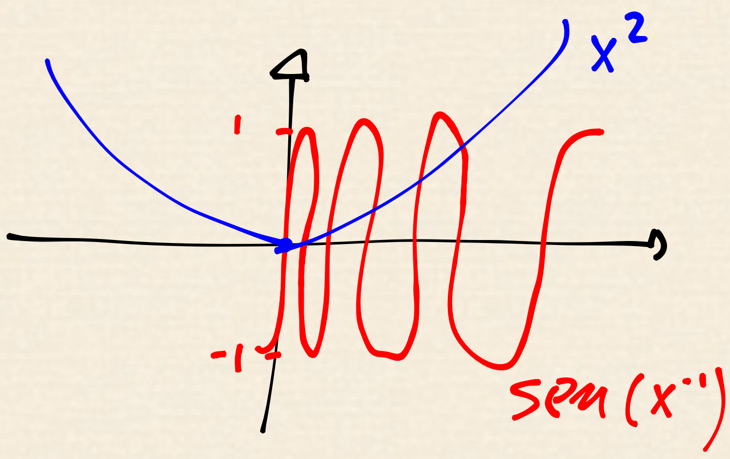
$$\frac{x+2}{\left(\sqrt{x^2+5}-3\right)} \cdot \frac{\left(\sqrt{x^2+5}+3\right)}{\left(\sqrt{x^2+5}+3\right)} = \frac{(x+2)\left(\sqrt{x^2+5}+3\right)}{\underbrace{x^2+5-9}} = \frac{\cancel{(x+2)}\left(\sqrt{x^2+5}+3\right)}{(x-2)\cancel{(x+2)}}$$

$\hookrightarrow = x^2-4 = (x-2)(x+2)$

$$\lim_{x \to -2} \frac{x+2}{\sqrt{x^2+5}-3} = \lim_{x \to -2} \frac{\sqrt{x^2+5}+3}{x-2} = \frac{\sqrt{4+5}+3}{-4} = -\frac{6}{4} = -\frac{3}{2}$$ //

19. $\lim\limits_{x \to 0} x^2 \operatorname{sen}(x^{-1})$, $\operatorname{sen}(x^{-1})$ é função limitada.

$\hookrightarrow |\operatorname{sen}(x^{-1})| \leq 1$



$$\lim_{x \to 0} x^2 \operatorname{sen}(x^{-1}) = \lim_{x \to 0} x^2 = 0$$

$$\lim_{x \to \infty} e^x \cdot \frac{1}{x} = \text{“}\infty \cdot 0\text{”} ?$$

$$= \infty$$

---

18. $\lim\limits_{x \to 0} \dfrac{\cos x - 1}{x}$, $\cos^2 x = 1 - \operatorname{sen}^2 x$

$$\frac{\cos x - 1}{x} \cdot \frac{\cos x + 1}{\cos x + 1} = \frac{\cos^2 x - 1}{x(\cos x + 1)} = \frac{-\operatorname{sen}^2 x}{x(\cos x + 1)}$$

$$= -\frac{\operatorname{sen} x}{x} \cdot \frac{\operatorname{sen} x}{\cos x + 1}$$

$$\Rightarrow \lim_{x \to 0} \frac{\cos x - 1}{x} = \lim_{x \to 0} \left(-\frac{\operatorname{sen} x}{x}\right)\left(\frac{\operatorname{sen} x}{\cos x + 1}\right) = -1 \cdot 0 = 0$$ //

LISTA, SEÇÕES 2 e 3.

SEC. 2, (2) $\lim_{x \to 1^+} f(x)$? $\lim_{x \to 1^-} f(x)$? , $f(x) = \dfrac{\sqrt{2x}\,(x-1)}{|x-1|}$

$$|x-1| = \begin{cases} x-1, & x-1 \geq 0 \to x \geq 1 \\ -(x-1), & x-1 < 0 \to x < 1 \end{cases}$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{\sqrt{2x}\,\cancel{(x-1)}}{\cancel{(x-1)}} = \lim_{x \to 1^+} \sqrt{2x} = \sqrt{2} \,//$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{\sqrt{2x}\,\cancel{(x-1)}}{-\cancel{(x-1)}} = -\sqrt{2} \,//$$

$$\lim_{x \to 1^+} f(x) \neq \lim_{x \to 1^-} f(x) \Rightarrow \lim_{x \to 1} f(x) \nexists.$$

(4) $f(x) = \dfrac{4-x}{x^2-2x-8}$, $\lim_{\substack{x \to 4^- \\ x \to 4^+}} f(x)$?

$$\begin{array}{r|l} x^2-2x-8 & -x+4 \\ \underline{-x^2+4x} & -x-2 \\ 2x-8 & \\ \underline{-2x+8} & \\ 0 & \end{array} \Rightarrow x^2-2x-8 = (-x+4)(-x-2)$$

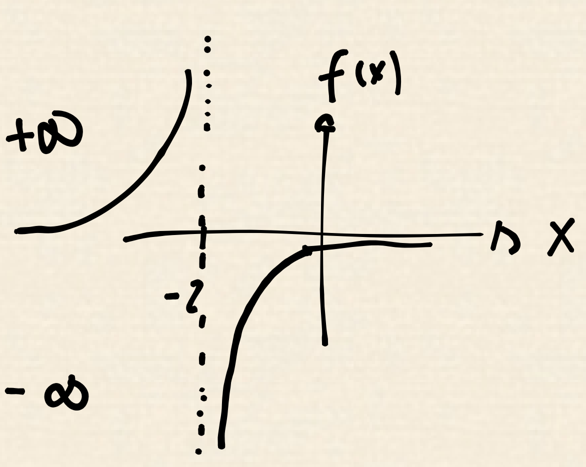
$$f(x) = \frac{\cancel{(4-x)}}{\cancel{(4-x)}(-x-2)} = \frac{1}{-x-2} = -\frac{1}{x+2}$$

$$\left.\begin{aligned} \lim_{x \to 4^+} f(x) &= \lim_{x \to 4^+} \frac{-1}{x+2} = -\frac{1}{6} \\ \lim_{x \to 4^-} f(x) &= \lim_{x \to 4^-} \frac{-1}{x+2} = -\frac{1}{6} \end{aligned}\right] \Rightarrow \lim_{x \to 4} f(x) = -\frac{1}{6}$$

Extra (Limites Infinitos)

$$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} \frac{-1}{x+2} = \text{“}\frac{-1}{0^-}\text{”} = +\infty$$

$$\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} \frac{-1}{x+2} = \text{“}\frac{-1}{0^+}\text{”} = -\infty$$



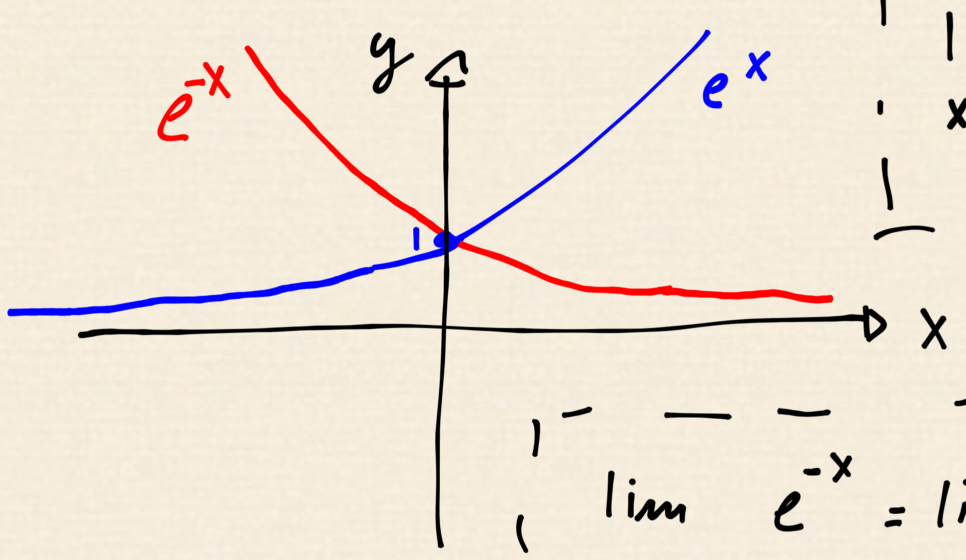
SEC. 3

(3) $$\lim_{x\to-\infty} \frac{2x+5}{\sqrt{2x^2-5}} = \lim_{x\to-\infty} \frac{\cancel{x}\,(2+5/x)}{-\cancel{x}\,\sqrt{2-5/x^2}} = -\frac{2}{\sqrt{2}} \quad //.$$

- - - - - - -

$$\sqrt{2x^2-5} = \sqrt{x^2(2-5/x^2)}$$
$$= \sqrt{x^2}\,\sqrt{2-5/x^2}$$
$$= |x|\,\sqrt{2-5/x^2} = -x\sqrt{2-\frac{5}{x^2}}$$ para $x<0$.

(4) $$\lim_{x\to 0^-} e^{1/x}$$



$$\lim_{x\to\infty} e^x = +\infty$$
$$\lim_{x\to-\infty} e^x = 0$$

Comportamento básico da função exponencial.

$$\lim_{x\to+\infty} e^{-x} = \lim_{x\to\infty} \frac{1}{e^x} = 0$$
$$\lim_{x\to-\infty} e^{-x} = \lim_{x\to-\infty} \frac{1}{e^x} = +\infty$$

$u = \frac{1}{x}$, $\lim_{x\to 0^-} u = -\infty$, $x \to 0^-$, $u \to -\infty$

$$\lim_{x\to 0^-} e^{1/x} = \lim_{u\to-\infty} e^u = 0 \quad //.$$

Dicas para (6) e (7)

$$\lim_{x\to\pm\infty} f(x)\,g(x) = \lim_{x\to\pm\infty} g(x)$$ se $f(x)$ é limitada.

Comentários do Teste 1.

a) $$\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} \cdot \frac{\sqrt{x}+4}{\sqrt{x}+4} = \lim_{x \to 16} \frac{\cancel{(x-16)}}{\cancel{(x-16)}(\sqrt{x}+4)} = \lim_{x \to 16} \frac{1}{\sqrt{x}+4}$$

$$u^2 = x \to \lim_{u \to 4} \frac{\sqrt{u^2} - 4}{u^2 - 16} = \lim_{u \to 4} \frac{\cancel{(u-4)}}{\cancel{(u-4)}(u+4)} = \lim_{u \to 4} \frac{1}{u+4}$$

$x \to 16$

$u \to 4$

b) $$\lim_{x \to 6} (x-6) \frac{\text{sen}(x+6)}{\text{sen}(x-6)} = \lim_{x \to 6} \frac{1}{\frac{\text{sen}(x-6)}{(x-6)}} \cdot \text{sen}(x+6)$$

$$= \lim_{u \to 0} \frac{1}{\underbrace{\frac{\text{sen}(u)}{u}}_{\to 1}} \cdot \lim_{x \to 6} \text{sen}(x+6)$$

$u = x - 6$

$x \to 6, \; u \to 0$

$$= \text{sen}(12)$$

VIDEO AULA 3

LIMITES INFINITOS

Quando uma função cresce indefinidamente para $x \to x_0$, podemos expressar tal comportamento por

$$\lim_{x \to x_0} f(x) = +\infty.$$

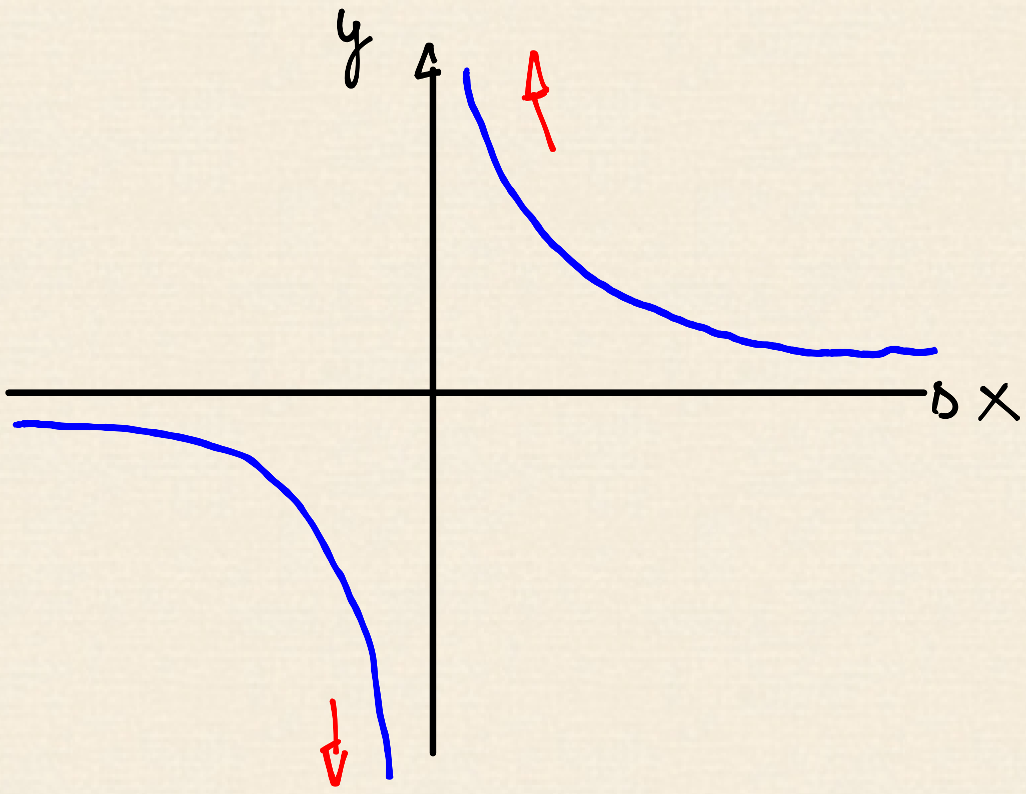
Analogamente, se decresce indefinidamente

$$\lim_{x \to x_0} f(x) = -\infty.$$

Ex. 1 $f(x) = \frac{1}{x}$

$$\lim_{x \to 0^+} f(x) = +\infty$$

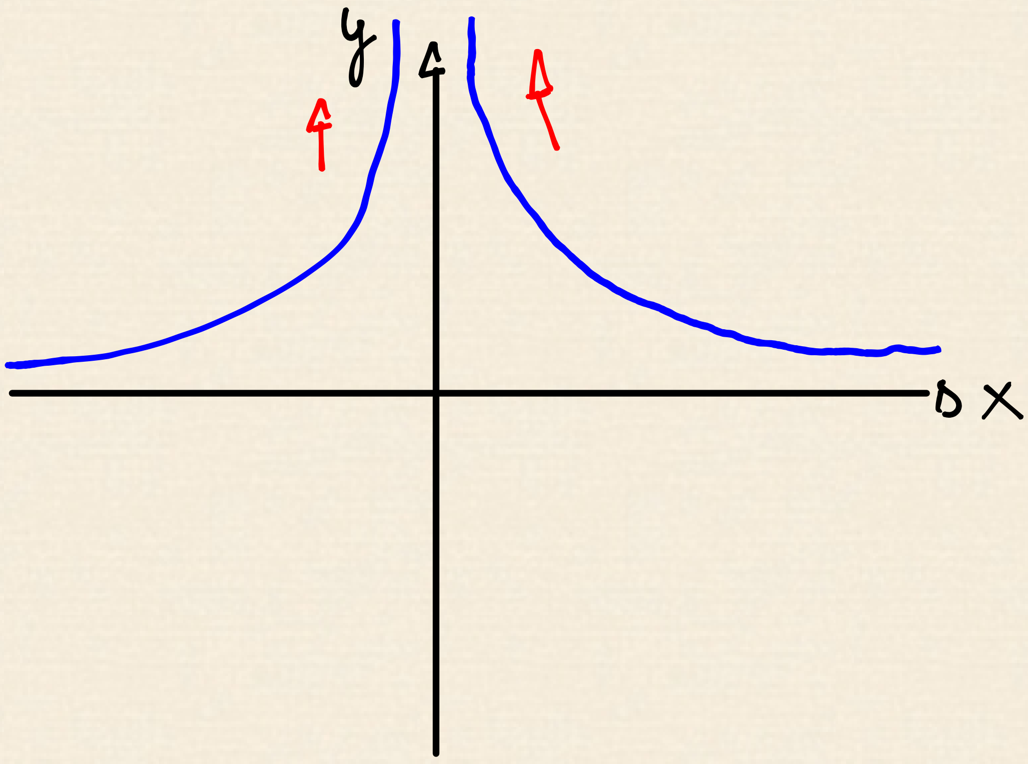
$$\lim_{x \to 0^-} f(x) = -\infty$$



Ex. 2 $f(x) = \frac{1}{x^2}$

$$\lim_{x \to 0^+} f(x) = +\infty$$

$$\lim_{x \to 0^-} f(x) = +\infty$$



ASSÍNTOTAS

A reta $y = b$ é uma assíntota horizontal da função $f(x)$ se:

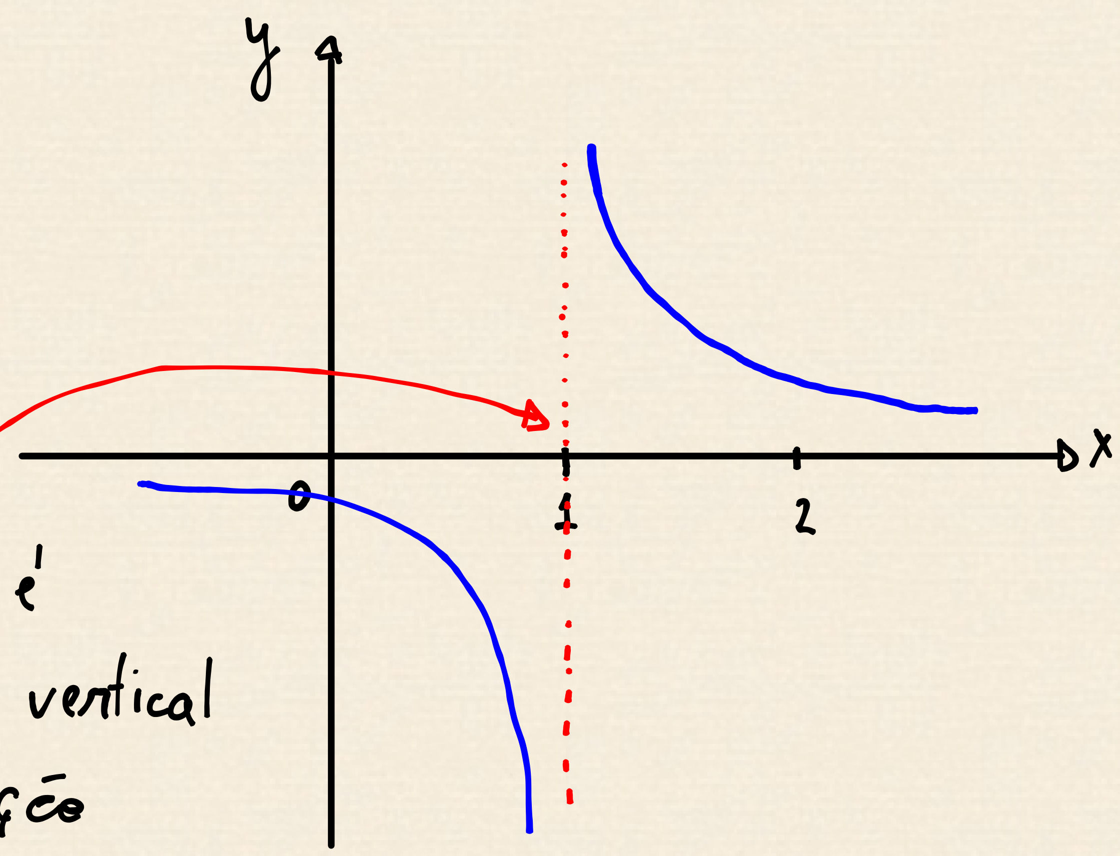
$$\lim_{x \to \infty} f(x) = b \quad \text{ou} \quad \lim_{x \to -\infty} f(x) = b.$$

A reta $x = a$ é uma assíntota vertical da função $f(x)$ se:

$$\lim_{x \to a^+} f(x) = \pm\infty \quad \text{ou} \quad \lim_{x \to a^-} f(x) = \pm\infty.$$

Ex. 1 $\quad f(x) = \dfrac{x}{x^2 - 1}$

$\lim_{x \to 1^-} f(x)$? $\quad \lim_{x \to 1^+} f(x)$?



$$\left.\begin{array}{l} \lim_{x \to 1^+} f(x) = +\infty \\ \lim_{x \to 1^-} f(x) = -\infty \end{array}\right\} \to$$

$x = 1$ é assíntota vertical da função

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \underbrace{\frac{x}{x^2}}_{\to 0} \cdot \frac{1}{1 - \underbrace{\frac{1}{x^2}}_{\to 0}} = 0$$

$$\lim_{x \to -\infty} f(x) = 0$$

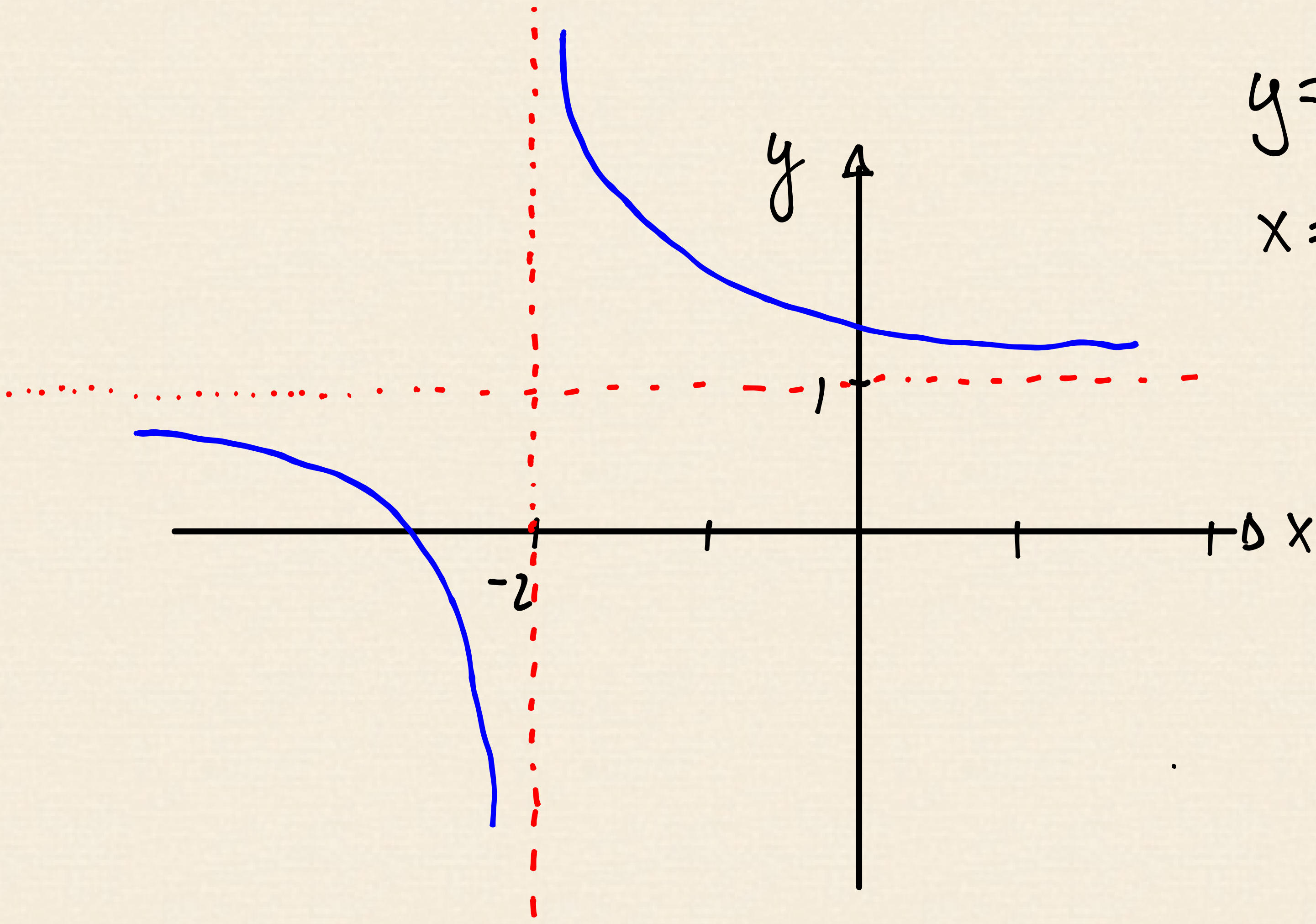
$\to$ $y = 0$ é assíntota horizontal da função.

Ex.2 $f = \frac{x+3}{x+2}$ Encontre as assíntotas.

Horizontais: $\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{x}{x} \frac{(1 + \cancel{3/x}^{\,0})}{(1 + \cancel{2/x}_{\,0})} = 1$ //

Verticais: $\lim_{x \to \underline{a}^{-,+}} f(x) = \pm\infty$?

$\lim_{x \to -2^-} \frac{x+3}{x+2} = -\infty$, $\lim_{x \to -2^+} \frac{x+3}{x+2} = +\infty$



$y = 1$ é A.H.

$x = -2$ é A.V.

VIDEO AULA 4.

CONTINUIDADE

$f(x)$ é contínua em $x = c$, se

$$\boxed{\lim_{x \to c} f(x) = f(c)}$$

Ex.1

$$f(x) = \frac{x^2-1}{x-1} \quad , \quad \lim_{x \to 1} f(x) = \text{"}\frac{0}{0}\text{"}$$

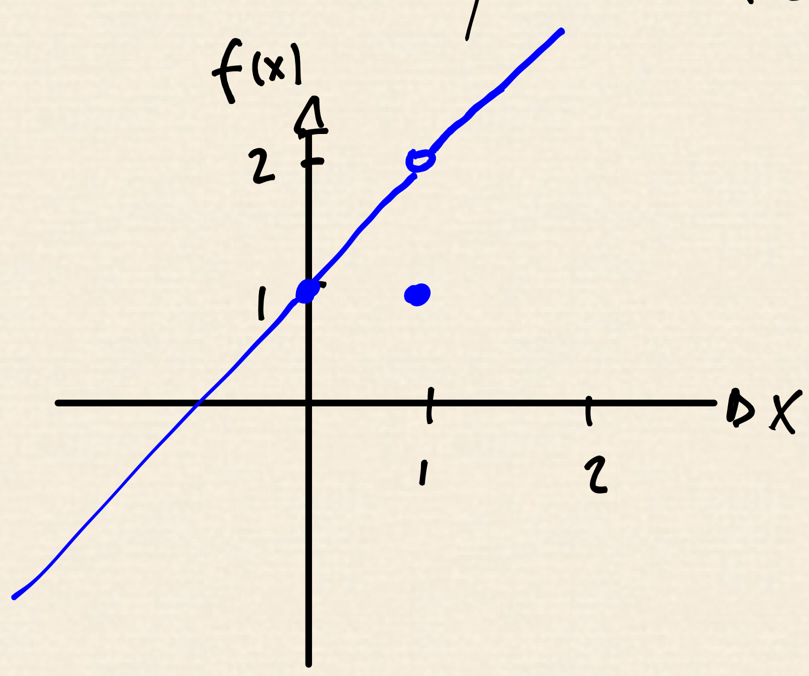
$$\lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = 2 \quad , \quad f(1) \nexists$$

Ex.2 $f(x) = \begin{cases} 1 & , \ x = 1 \\ \dfrac{x^2-1}{x-1} & , \ x \in (-\infty, 1) \cup (1, +\infty) \end{cases}$

$\lim_{x \to 1} f(x) = 2 \neq f(1) = 1 \Rightarrow f$ não é contínua em $x = 1$.

$\lim_{x \to c} f(x) = f(c)$ para $c \in (-\infty, 1) \cup (1, +\infty)$

$\Rightarrow f$ é contínua para $x \in \mathbb{R}$, $x \neq 1$.



Ex.3

$$f(x) = \begin{cases} -2 & , -2 < x < -1 \\ -x^2+x & , -1 < x < 0 \\ x & , 0 \leq x < 1 \\ \sqrt{x}+1 & , x \geq 1 \end{cases}$$

Continuidade: $x=-1$, $f(-1) \nexists$.

$$\lim_{x\to -1} f(x) \begin{cases} \lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} (-2) = -2 \\ \lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} (-x^2+x) = -2 \end{cases}$$

$\Rightarrow \lim_{x\to -1} f(x) = -2$ pois $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x) = -2$

$x=0$

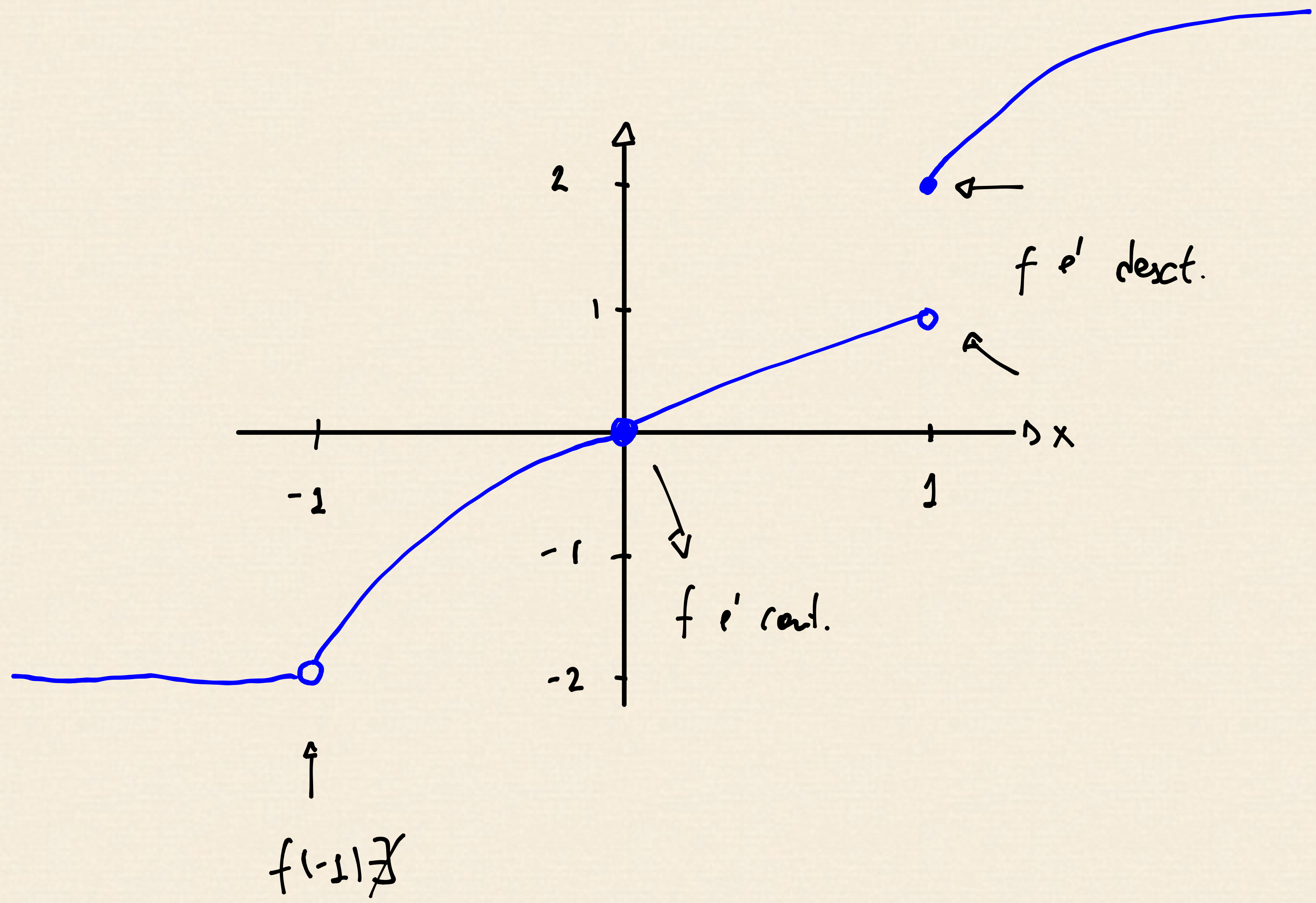
$$\lim_{x\to 0} f(x) \begin{cases} \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (-x^2+x) = 0 \\ \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x = 0 \end{cases} \Rightarrow \lim_{x\to 0} f(x) = 0$$

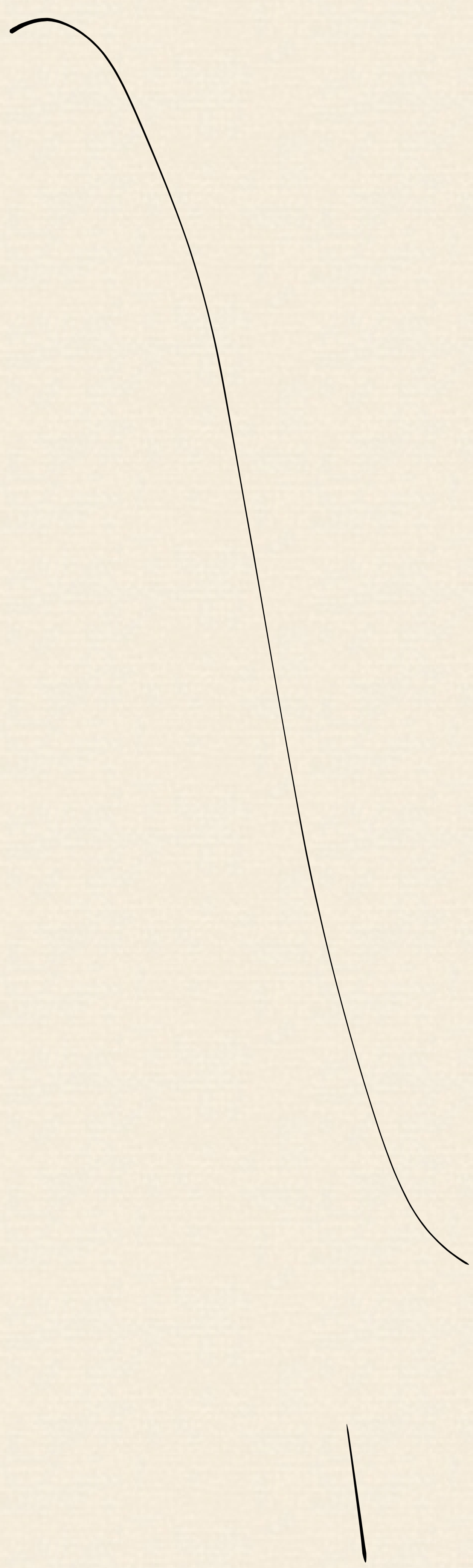
$f(0) = x\Big|_{x=0} = 0$, $\lim_{x\to 0} f(x) = f(0) \Rightarrow f$ é contínua em $x=0$.

$x=1$

$$\lim_{x\to 1} f(x) \begin{cases} \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x = 1 \\ \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (\sqrt{x}+1) = 2 \end{cases} \Rightarrow \lim_{x\to 1} f(x) \nexists$$

f é descontínua em $x=1$.


2
1
-1
-2
-1
1
x
f e' dext.
f e' cont.
f(-1) ∄



Lista 1, Sec. 4

(2) $f(x) = \dfrac{3-x}{x-1}$

Assíntotas Horizontais: $\lim\limits_{x \to \pm\infty} f(x)$?

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{\cancel{x}}{\cancel{x}} \frac{(\overbrace{3/x}^{\to 0} - 1)}{(1 - \underbrace{1/x}_{\to 0})} = -1 \Rightarrow y = -1$$

é A.H.

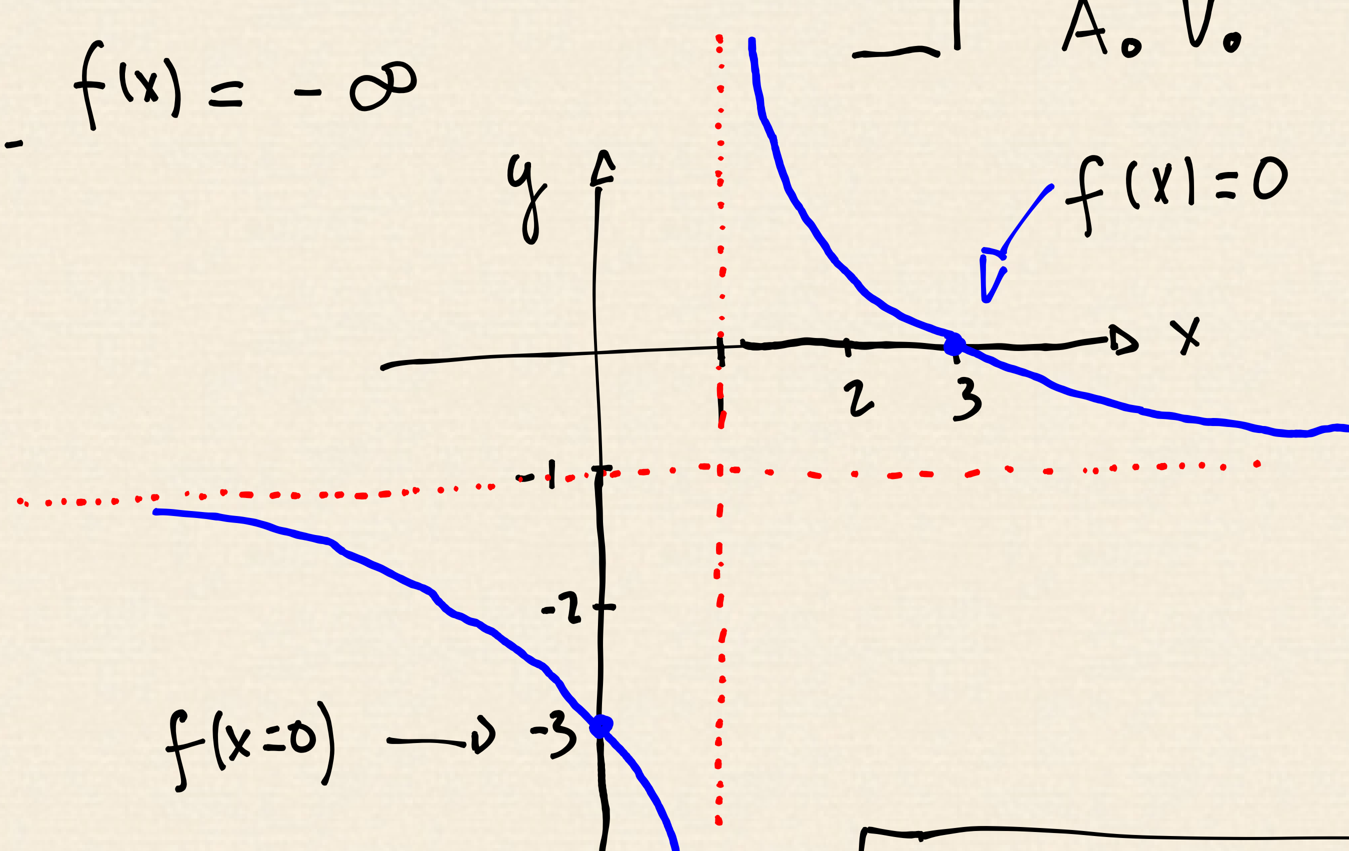
Assíntotas Verticais:

$$\lim_{x \to a^{+,-}} f(x) = \pm\infty$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{3-x}{x-1} = +\infty$$

$$\lim_{x \to 1^-} f(x) = -\infty$$

$x = 1$ é A. V.



$$f(x) = 0 \Rightarrow \frac{3-x}{x-1} = 0 \Rightarrow x = 3$$

$$f(x=0) = \frac{3}{-1} = -3$$

$$\exp(f(x)) = e^{f(x)}$$

$$\exp(x^2) = e^{x^2}$$

$$\exp\left(\frac{ax^2+bx+c}{dx^2+ex+f}\right) = e^{\frac{ax^2 \ldots}{\ldots}}$$

Sec. 5

(3) $f(x) = \begin{cases} \dfrac{x^2-1}{x^2+x-2}, & x < 1 \\ -x^3+x-2, & 1 \leq x \leq 2 \\ \exp\left(\dfrac{x^2-4}{x^2-3x+2}\right) & x > 2 \end{cases}$

Continuidade em $x=1$: $\lim\limits_{x \to 1} f(x) = f(1)$?

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{x^2-1}{x^2+x-2} = \lim_{x \to 1^-} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(x+2)} = \frac{2}{3}$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (-x^3+x-2) = -2$$

$$\lim_{x \to 1^+} f(x) \neq \lim_{x \to 1^-} f(x) \Rightarrow \lim_{x \to 1} f(x) \nexists .$$

Então f é descontínua em $x=1$.

Continuidade em $x=2$: $\lim\limits_{x \to 2} f(x) = f(2)$?

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \exp\left(\frac{x^2-4}{x^2-3x+2}\right)$$

$$\lim_{x \to 2^+} \underbrace{\left(\frac{x^2-4}{x^2-3x+2}\right)}_{=u} = \lim_{x \to 2^+} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x-1)} = 4 \qquad \begin{array}{l} x \to 2^+ \\ u \to 4 \end{array}$$

$$\lim_{x \to 2^+} f(x) = \lim_{u \to 4} e^u = e^4$$

$x^2-4 \ \underline{|x-2}$
$\vdots \qquad x+2$

$x^2-3x+2 \ \underline{|x-2}$
$\vdots \qquad x-1$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (-x^3+x-2) = -8+2-2=-8$$

$$\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x) \Rightarrow \lim_{x\to 2} f(x) \nexists$$

Então f é descontínua em $x=2$.

SEC. 2

① $f(x) = \begin{cases} -x, & x<0 \\ 1, & 0<x<1 \\ x^2, & x \geq 1 \end{cases}$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (-x) = 0, \quad \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 1 = 1$$

$$\Rightarrow \lim_{x\to 0} f(x) \nexists$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 1 = 1, \quad \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2 = 1$$

$$\Rightarrow \lim_{x\to 1} f(x) = 1.$$

SEC. 3

⑤ $$\lim_{x\to -\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} = \lim_{x\to -\infty} \frac{\cancel{e^{-x}}}{\cancel{e^{-x}}} \cdot \frac{\overbrace{e^{2x}}^{\to 0} - 1}{\underbrace{e^{2x}}_{\to 0} + 1} = -1$$

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$$\lim_{x\to -\infty} \frac{x^2 - x^{-2}}{x^2 + x^{-2}}, \quad x^{-2} = 1/x^2$$

$$e^{-x} = 1/e^x$$

SEC. 2, 3 $\lim_{x \to -2^{+,-}} \frac{|x+2|}{x^2-4}$

$|x+2| = -(x+2)$, $x < -2$ (esquerda de -2).

$$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} \frac{-(x+2)}{x^2-4} = \lim_{x \to -2^-} -\frac{\cancel{(x+2)}}{(x-2)\cancel{(x+2)}} = \lim_{x \to -2^-} \frac{-1}{x-2}$$

$$\lim_{x \to -2^-} f(x) = \frac{1}{4} \quad //$$