17/4/2021

Integrais Indefinidas

F é primitiva de f se

$$\frac{d}{dx} F(x) = f(x)$$

Ex.1 $f(x) = x^2$

$$\frac{d}{dx} x^3 = 3x^2 \Rightarrow \frac{d}{dx}\left(\frac{x^3}{3}\right) = x^2$$

$$F(x) = \frac{x^3}{3} + K$$

Ex.2 $f(x) = \cos(2x)$

$$\frac{d}{dx} \operatorname{sen}(2x) = 2\cos(2x) \Rightarrow \frac{d}{dx}\left(\frac{\operatorname{sen}(2x)}{2}\right) = \cos(2x)$$

$$F(x) = \frac{\operatorname{sen}(2x)}{2} + K$$

Ex.3 $f(x) = \frac{2}{x}$

$$\frac{d}{dx} \ln(x) = \frac{1}{x} \Rightarrow \frac{d}{dx}\left(2\ln(x)\right) = \frac{2}{x}$$

$$F(x) = 2\ln(x) + K$$

$$F(x) = \ln x^2 + K$$

$$\frac{d}{dx} \ln x^2 = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$$

Ex. 4 $\quad f(x) = 2x\, e^{x^2}$

$$\frac{d}{dx} e^{x^2} = 2x\, e^{x^2} \quad , \quad F(x) = e^{x^2} + K$$

$$\boxed{\begin{array}{c}\text{Integral Indefinida} \\ \int f(x)\, dx = F(x) + K\end{array}}$$

$$\boxed{\begin{array}{l}\sum f(x_i)\, \Delta x_i \\ \to \int f(x)\, dx\end{array}}$$

Integral Definida $\quad \displaystyle\int_a^b f(x)\, dx = F(b) - F(a)$

Relação da Int. Indefinida com a Derivada:

$$\frac{d}{dx}\left(F(x) + K\right) = f(x)$$

$$\frac{d}{dx}\left[\int f(x)\, dx\right] = f(x) = \int \frac{df}{\cancel{dx}}\, \cancel{dx} = \boxed{\int df = f(x)}$$

A Int. Ind. é a operação inversa da Derivada

e vice-versa.

$$\boxed{\frac{d}{dx}\int f(x)\, dx = \int dx \left(\frac{df}{dx}\right)}$$

Ex. $\quad \displaystyle\int dx \left(\frac{d}{dx} \sec(x)\right) = \int \sec(x) \cdot tg(x)\, dx$

$$\int d \sec(x) = \boxed{\sec(x) = \int \sec(x) \cdot tg(x)\, dx}$$

Propriedades da Integral

$$\int k\, f(x)\, dx = k \int f(x)\, dx$$

$$\int (f(x) + g(x))\, dx = \int f(x)\, dx + \int g(x)\, dx$$

A integral é um operador linear.

Integrais Imediatas

$$\frac{d}{dx}(x) = 1 \rightarrow \int 1\, dx = x + C$$

$$\frac{d}{dx} x^b = b\, x^{b-1},\ b-1 = a \rightarrow \int x^a\, dx = \frac{x^{a+1}}{a+1} + C,\ a \neq -1.$$

$$\frac{d}{dx} e^x = e^x \rightarrow \int e^x dx = e^x + C$$

$$\frac{d}{dx} \ln|x| = \frac{1}{x} \rightarrow \int \frac{1}{x}\, dx = \ln|x| + C$$

$$\frac{d}{dx} \operatorname{sen}(x) = \cos x \rightarrow \int \cos x\, dx = \operatorname{sen} x + C$$

$$\frac{d}{dx} \cos(x) = -\operatorname{sen}(x) \rightarrow \int \operatorname{sen}(x)\, dx = -\cos(x) + C$$

19/04/2021

INTEGRAÇÃO POR MUDANÇA DE VARIÁVEL

$$\frac{d}{dx} F(x) = f(x)$$

$$\frac{d}{du} F(u) = f(u)$$

Pela regra da cadeia:

$$\frac{d}{dx} F(u(x)) = \frac{dF}{du}\frac{du}{dx} = f(u)\frac{du}{dx}$$

$$\int f(u)\frac{du}{dx}\,dx = \int f(u)\,du = F(u) + C$$

Ex.0

$$\int \cos(x^2)\,\underline{2x\,dx} = \operatorname{sen}(x^2) + K$$

$$\frac{d}{dx}\operatorname{sen}(x^2) = 2x\cos(x^2)$$

$$u = x^2$$

$$\frac{du}{dx} = 2x \Rightarrow du = \underline{2x\,dx}$$

$$\int \cos(x^2)\,2x\,dx = \int \cos(u)\,du = \operatorname{sen}(u) + K = \operatorname{sen}(x^2) + K \;//$$

Ex.1 $$\int \frac{2x}{x^2+1}\,dx = \int \frac{du}{u} = \ln|u| + K = \ln|x^2+1| + K \;//$$

$$u = x^2 + 1$$

$$du = 2x\,dx$$

Ex.2 $\int \operatorname{sen}^2 x \cos x \, dx = \int u^2 \, du = \frac{u^3}{3} + K$

$u = \operatorname{sen} x$ $\qquad = \frac{\operatorname{sen}^3 x}{3} + K$ //

$du = \cos x \, dx$

Ex.3 $\int \operatorname{sen}(2x+8) \, dx = \int \operatorname{sen}(u) \frac{du}{2} = -\frac{\cos(u)}{2} + K$

$u = 2x+8$ $\qquad = -\frac{\cos(2x+8)}{2} + K$

$\frac{du}{dx} = 2 \rightarrow dx = \frac{du}{2}$

Ex.4 $\int tg(x) \, dx = \int \frac{\operatorname{sen}(x)}{\cos(x)} dx = \int -\frac{du}{u} = -\ln|u| + K$

$u = \cos(x)$ $\qquad = -\ln|\cos(x)| + K$ //

$du = -\operatorname{sen}(x) \, dx$

Ex.5 $\int \frac{1}{(2x+8)^2} dx = \int \frac{du/2}{u^2} = \frac{1}{2} \int u^{-2} du$

$u = 2x+8$ $\qquad = \frac{1}{2} \frac{u^{-1}}{(-1)} = -\frac{1}{2} \frac{1}{u}$

$du = 2 \, dx$

$= -\frac{1}{4x+16} + K$ //

Ex.6 $\int x \exp\left(\frac{x^2}{3} - 1\right) dx = \int \frac{3}{2} du \, e^u = \frac{3}{2} e^u + K$

$u = \frac{x^2}{3} - 1$ $\qquad = \frac{3}{2} \exp\left(\frac{x^2}{3} - 1\right) + K$ //

$du = \frac{2}{3} x \, dx \rightarrow x \, dx = \frac{3}{2} du$

20/4/21 Seção 1 da Lista.

① $\int (x^4 + 2x^3 - 2x + a)\,dx = \int x^4\,dx + 2\int x^3\,dx - 2\int x\,dx + a\int dx$

$= \frac{x^5}{5} + 2\frac{x^4}{4} - 2\frac{x^2}{2} + ax + C$

$= \frac{x^5}{5} + \frac{x^4}{2} - x^2 + ax + C$ //

② $\int \left(x^3 - \frac{\sqrt{x}}{3} + \frac{2}{x} \right) dx = \int x^3\,dx - \frac{1}{3}\int x^{1/2}\,dx + 2\int \frac{dx}{x}$

$= \frac{x^4}{4} - \frac{1}{\cancel{3}} \frac{x^{3/2}}{\cancel{3}/2} + 2\ln|x| + C$

$= \frac{x^4}{4} - 2x^{3/2} + 2\ln|x| + C$ //

22/4/2021

③ $A = \int \left(e^{ax} + b\cos(2x) \right) dx = \underbrace{\int e^{ax}\,dx}_{\mathrm{I}} + b\underbrace{\int \cos(2x)\,dx}_{\mathrm{II}}$

I $u = ax$

$\frac{du}{dx} = a \Rightarrow dx = \frac{1}{a}du \Rightarrow \mathrm{I} = \int e^u \frac{du}{a} = \frac{1}{a}e^u = \frac{1}{a}e^{ax}$

$\frac{1}{a}\frac{d}{dx}e^{ax} = \frac{1}{\cancel{a}}\cancel{a}e^{ax} = e^{ax}$

II $\int \cos(2x)\,dx = \int \cos(u)\frac{du}{2} = \frac{\mathrm{sen}(u)}{2} = \frac{\mathrm{sen}(2x)}{2}$

$u = 2x$

$\frac{du}{dx} = 2 \Rightarrow dx = \frac{1}{2}du$

$A = \frac{e^{ax}}{a} + \frac{b}{2}\mathrm{sen}(2x) + K.$ //

④ $A=\int\left(\frac{\operatorname{sen}(x)}{\cos^2(x)}+\frac{2}{x^7}\right)dx=\underbrace{\int\frac{\operatorname{sen}x\,dx}{\cos^2x}}_{I}+2\int x^{-7}dx$

I: $u=\cos(x)$

$\frac{du}{dx}=-\operatorname{sen}(x) \Rightarrow du=-\operatorname{sen}x\,dx$

$I=-\int\frac{du}{u^2}=-\frac{u^{-1}}{(-1)}=\frac{1}{\cos x}$

Verificando: $\frac{d}{dx}(\cos x)^{-1}=-1(\cos x)^{-2}(-\operatorname{sen}x)=\frac{\operatorname{sen}x}{\cos^2x}$

$\Rightarrow A=\frac{1}{\cos x}+2\frac{x^{-6}}{-6}+K=\frac{1}{\cos x}-\frac{x^{-6}}{3}+K$ //

⑥ $\int\frac{x^2}{x^2+1}dx=\int 1\,dx-\int\frac{dx}{x^2+1}=x-\operatorname{arctg}(x)+K$ //

$\frac{x^2}{x^2+1}=\frac{x^2+1-1}{x^2+1}=1-\frac{1}{x^2+1}$

$\boxed{\frac{d}{dx}\operatorname{arctg}(x)=\frac{1}{1+x^2}}$

⑦ $\int\sqrt{\frac{4a^2}{x^4-x^2}}\,dx\ \ldots$

$\sqrt{\frac{4a^2}{x^4-x^2}}=\frac{2a}{\sqrt{x^2(x^2-1)}}=\frac{2a}{|x|\sqrt{x^2-1}}$ - ver derivada de arcsec(x).

⑧ $\int\frac{x}{2}\operatorname{sen}(3x^2+2)dx=\frac{1}{2}\cdot\frac{1}{6}\int\operatorname{sen}u\,du=\ldots$

$u=3x^2+2$

$du=6x\,dx$

⑨ $\int\frac{dt}{t^2+a^2}=\frac{1}{a^2}\int\frac{dt}{1+(t/a)^2}=\ldots$

$u=t/a\ \ldots$

$\cancel{10}$ $\displaystyle\int \frac{dx}{x^2+4x+7} = \int \frac{A\,dx}{(x-x_1)} + \int \frac{B\,dx}{(x-x_2)}$?

$$\frac{d}{dx}\ln(ax+b) = \frac{a}{ax+b}$$

$x^2+4x+7=0$

$x = \frac{1}{2}\left[-4 \pm \sqrt{16-28}\right]$

Não fazer o exercício (10).

(11) $\displaystyle\int \sqrt{t^2-2t^4}\,dt \underset{t>0}{=} \int t\sqrt{1-2t^2}\,dt = \int \frac{du}{-4} u^{1/2} = \frac{u^{3/2}}{-4\cdot 3/2} + k$

$\sqrt{t^2(1-2t^2)} = |t|\sqrt{1-2t^2}$

$= -\dfrac{(1-2t^2)^{3/2}}{6} + k$ //

$u = 1-2t^2$

$du = -4t\,dt$

(12) $\displaystyle\int \frac{e^x}{e^x+4}\,dx = \int \frac{du}{u} = \dots$

$u = e^x+4$

$du = e^x\,dx$

(14) $\displaystyle\int \frac{\ln(x^2)}{x}\,dx = 2\int \frac{\ln x}{x}\,dx = 2\int u\,du = u^2 = (\ln x)^2 + k$ //

$u = \ln x$

$du = \frac{1}{x}\,dx$

(15) Desafio: $\displaystyle\int \frac{\sqrt{x+3}}{x-1}\,dx$, Dica: $u = \sqrt{x+3}$

(16) $\displaystyle\int x^2\sqrt{1+x}\,dx = \int (u-1)^2 u^{1/2}\,du = \dots$

$u = 1+x \rightarrow (u-1)^2 = x^2$

$du = dx$

(18) não cai no teste!

INTEGRAÇÃO POR PARTES

$$f(x) = u(x) \cdot v(x)$$

$$\frac{df}{dx} = \frac{du}{dx} v + u \frac{dv}{dx}$$

$$\int dx \frac{df}{dx} = \int dx \frac{du}{dx} v + \int dx\, u \frac{dv}{dx}$$

$$\int df = \int v\, du + \int u\, dv$$

$$uv = \int v\, du + \int u\, dv$$

$$\boxed{\int u\, dv = uv - \int v\, du}$$

Ex.1

$$\int x e^{-2x} dx = x \frac{e^{-2x}}{(-2)} - \int \frac{e^{-2x}}{(-2)} dx$$

$u = x$; $dv = e^{-2x} dx$

$du = dx$; $v = \int e^{-2x} dx$

$v = \dfrac{e^{-2x}}{-2}$

$$= -\frac{1}{2} x e^{-2x} + \frac{1}{2} \int e^{-2x} dx$$

$$= -\frac{1}{2} x e^{-2x} + \frac{1}{2} \frac{e^{-2x}}{(-2)} + K$$

$$= \frac{e^{-2x}}{2} \left(-x - \frac{1}{2} \right) + K = F(x)$$

$$\frac{d}{dx} F(x) = \frac{d}{dx} \left[-\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} \right] = \cancel{-\frac{1}{2} e^{-2x}} + x e^{-2x} + \cancel{\frac{1}{2} e^{-2x}}$$

$$= x e^{-2x} \checkmark$$

Ex.2 $$\int \ln(x)\,dx = x\ln x - \int \cancel{x}\cdot\frac{1}{\cancel{x}}\,dx = x\ln x - x + K$$

$$u = \ln x \qquad dv = dx$$
$$du = \frac{1}{x}\,dx \qquad v = x$$

$$\frac{d}{dx}\left[x\ln x - x\right] = \ln x + \cancel{x\cdot\frac{1}{x}} - \cancel{1} = \ln x \checkmark$$

Ex.3 $$\int x^2 \,\text{sen}(x)\,dx = -x^2\cos x - \int 2x(-\cos x)\,dx$$

1ª $$u = x^2 \qquad dv = \text{sen}\,x\,dx$$
$$du = 2x\,dx \qquad v = -\cos x$$

2ª $$u = x \qquad dv = \cos x\,dx$$
$$du = dx \qquad v = \text{sen}\,x$$

$$= -x^2\cos x + 2\int x\cos x\,dx$$

$$= -x^2\cos x + 2\left[x\,\text{sen}\,x - \int \text{sen}\,x\,dx\right]$$

$$= -x^2\cos x + 2x\,\text{sen}\,x + 2\cos x + K$$

$$= \cos x(2 - x^2) + 2x\,\text{sen}\,x + K$$

Verificando: $$\frac{d}{dx}\left[\cos x(2-x^2) + 2x\,\text{sen}\,x\right]$$

$$= -\text{sen}\,x(\cancel{2}-x^2) + \cancel{\cos x(-2x)} + \cancel{2\,\text{sen}\,x} + \cancel{2x\cos x}$$

$$= x^2\,\text{sen}\,x \checkmark$$

TÉCNICAS DE INTEGRAÇÃO

Como integrar $\int \text{sen}^n(x)dx$ e $\int \cos^n(x)dx$?

n = inteiro.

Identidade trigonométricas úteis:

$$\boxed{\text{sen}^2 x + \cos^2 x = 1}$$

$$\boxed{\text{sen}^2 x = \frac{1}{2} - \frac{\cos(2x)}{2}} \quad , \quad \boxed{\cos^2 x = \frac{1}{2} + \frac{\cos(2x)}{2}}$$

Ex.1 $\int \cos^5 x\, dx = \int \cos^4 x \cos x\, dx = I$

$$\cos^4 x = (\cos^2 x)^2 = (1 - \text{sen}^2 x)^2 = 1 - 2\,\text{sen}^2 x + \text{sen}^4 x$$

$$I = \int (1 - 2\,\text{sen}^2 x + \text{sen}^4 x)\cos x\, dx$$

$$= \int \cos x\, dx - 2\int \text{sen}^2 x \cos x\, dx + \int \text{sen}^4 x \cos x\, dx$$

$$\boxed{\int \text{sen}^n x \cos x\, dx = \int u^n du = \frac{u^{n+1}}{n+1} = \frac{\text{sen}^{n+1}(x)}{n+1}}$$

$u = \text{sen}\, x$

$du = \cos x\, dx$

$$I = \text{sen}\, x - \frac{2}{3}\text{sen}^3 x + \frac{1}{5}\text{sen}^5(x) + K \quad //.$$

Fazer Ex.2 $\int \text{sen}^3 x\, dx$

Ex. 3 $\int sen^4 x \, dx = I$

$$\left(sen^2(x)\right)^2 = \left(\frac{1}{2} - \frac{cos(2x)}{2}\right)^2 = \frac{1}{4} - \frac{1}{2}cos(2x) + \frac{cos^2(2x)}{4}$$

$$sen^4(x) = \frac{1}{4} - \frac{1}{2}cos(2x) + \frac{1}{4}\left[\frac{1}{2} + \frac{1}{2}cos(4x)\right]$$

$$= \frac{3}{8} - \frac{1}{2}cos(2x) + \frac{1}{8}cos(4x)$$

$\left| \frac{1}{4} + \frac{1}{8} = \frac{3}{8} \right.$

$$\Rightarrow I = \int \left(\frac{3}{8} - \frac{1}{2}cos(2x) + \frac{1}{8}cos(4x)\right) dx$$

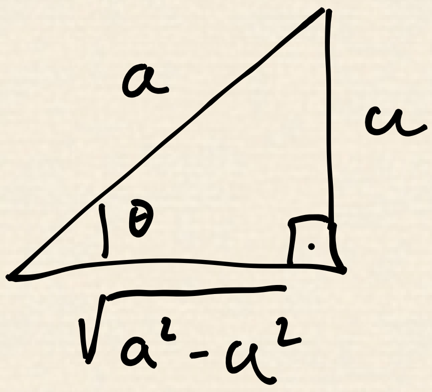
$$I = \frac{3}{8}x - \frac{1}{2}\int cos(2x)\,dx + \frac{1}{8}\int cos(4x)\,dx$$

$$= \frac{3}{8}x - \frac{1}{2}\,\frac{sen(2x)}{2} + \frac{1}{8}\,\frac{sen(4x)}{4} + K$$

$$I = \frac{3}{8}x - \frac{1}{4}sen(2x) + \frac{1}{32}sen(4x) + K \quad //$$

$$\frac{dI}{dx} = \frac{3}{8} - \frac{1}{2}cos(2x) + \frac{1}{8}cos(4x) = sen^4(x) \; \checkmark$$

SUBSTITUIÇÃO TRIGONOMÉTRICA



Quando houver funções envolvendo $\sqrt{a^2-u^2}$ numa integral, pode-se utilizar a seguinte substituição:

$$\boxed{u = a \operatorname{sen}\theta}$$

Ex.1 $\int \sqrt{4-x^2}\, dx = I$

$x = 2\operatorname{sen}\theta$, $\sqrt{4-x^2} = \sqrt{4-4\operatorname{sen}^2\theta} = 2\sqrt{1-\operatorname{sen}^2\theta}$

$dx = 2\cos\theta\, d\theta$ $\qquad = 2\cos\theta$

$$\Rightarrow I = \int 2\cos\theta\,(2\cos\theta)\,d\theta = 4\int \cos^2\theta\, d\theta$$

$$= 4\int d\theta \left[\frac{1}{2} + \frac{1}{2}\cos(2\theta)\right] = 4\left[\frac{\theta}{2} + \frac{1}{2}\frac{\operatorname{sen}(2\theta)}{2}\right] + K$$

$$\Rightarrow I = 2\theta + \operatorname{sen}(2\theta) + K, \quad x = 2\operatorname{sen}\theta$$

$$\Rightarrow \theta = \operatorname{arcsen}(x/2)$$

$$I = 2\operatorname{arcsen}(x/2) + \operatorname{sen}(2\operatorname{arcsen}(x/2)) + K \quad //$$

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$$\int \sqrt{u}\, dx = \int \sqrt{u}\, \frac{du}{-2x} = \int \frac{\sqrt{u}}{-2\sqrt{1-u}}\, du \ ?$$

$u = 1-x^2$

$du = -2x\,dx$ $\Rightarrow x^2 = 1-u \Rightarrow |x| = \sqrt{1-u}$

Exercícios da Lista, Seção 2

Integração por Partes: $\int u\,dv = uv - \int v\,du$

① $\int x e^{-x} dx = -xe^{-x} - \int (-e^{-x})\,dx = -xe^{-x} - e^{-x} = -e^{-x}(x+1) + K$

$u = x \quad \vdots \quad dv = e^{-x}dx$

$du = dx \quad \vdots \quad v = -e^{-x}$

④ $\underbrace{\int e^{2x}\,\text{sen}(x)\,dx}_{I} = -e^{2x}\cos x + \int 2\cos x\, e^{2x}dx$

$u = e^{2x} \quad \vdots \quad dv = \text{sen}(x)\,dx$

$du = 2e^{2x}dx \quad \vdots \quad v = -\cos x$

$= -e^{-2x}\cos x + 2\left[e^{2x}\,\text{sen}\,x - 2\underbrace{\int \text{sen}\,x\, e^{2x}dx}_{I}\right]$

2ª Integração por partes

$u = e^{2x} \quad \vdots \quad dv = \cos x\,dx$

$du = 2e^{2x}dx \quad \vdots \quad v = \text{sen}\,x$

$\Rightarrow I = -e^{-2x}\cos x + 2e^{2x}\,\text{sen}\,x - 4I$

$\Rightarrow 5I = e^{2x}(2\,\text{sen}\,x - \cos x)$

$I = \dfrac{e^{2x}}{5}(2\,\text{sen}\,x - \cos x) + K$

⑤ $\int \text{sen}^3(x)\,dx = I$

~~$u = \text{sen}^3 x \quad \vdots \quad dv = dx$~~

~~$du = 3\,\text{sen}^2 x\cos x\,dx \quad \vdots \quad v = x$~~

$u = \text{sen}^2 x \quad \vdots \quad dv = \text{sen}\,x\,dx$

$du = 2\,\text{sen}\,x\cos x\,dx \quad \vdots \quad v = -\cos x$

$I = -\text{sen}^2 x\cos x + 2\int \cos x(\text{sen}\,x\cos x)\,dx$

$I = -\text{sen}^2 x\cos x + 2\int \cos^2 x\,\text{sen}\,x\,dx$

Para 2ª integral

$u = \cos x$

$du = -\text{sen}\,x\,dx$

$I = -\text{sen}^2 x\cos x + 2\int u^2(-du)$

$I = -\text{sen}^2 x\cos x - 2\dfrac{u^3}{3} + K$

$I = -\text{sen}^2 x\cos x - \dfrac{2}{3}\cos^3 x + K$

SEC. 3

1. (a) $I = \int sen^3(x)\,dx = \int sen^2 x \; sen\,x\,dx = \int (1-\cos^2 x)\, sen\,x\,dx$

$$I = \int sen\,x\,dx - \underbrace{\int \cos^2 x \; sen\,x\,dx}_{\text{mesma feita no exercício anterior.}} = -\cos x + \frac{\cos^3 x}{3} + K \quad //$$

Verificar se esta resposta é equivalente à anterior.

1. (c) $I = \int \cos^4(x+1)\,dx$

$$\cos^2 x = \frac{1}{2} + \frac{\cos(2x)}{2}, \quad \cos^4(x+1) = \left(\frac{1}{2} + \frac{\cos(2x+2)}{2}\right)^2$$

$$= \frac{1}{4} + \frac{1}{2}\cos(2x+2) + \frac{\cos^2(2x+2)}{4}$$

$$\Rightarrow \cos^4(x+1) = \frac{1}{4} + \frac{1}{2}\cos(2x+2) + \frac{1}{4}\left(\frac{1}{2} + \frac{1}{2}\cos(4x+4)\right)$$

$$= \frac{3}{8} + \frac{1}{2}\cos(2x+2) + \frac{1}{8}\cos(4x+4)$$

$$\Rightarrow I = \int \frac{3}{8}\,dx + \frac{1}{2}\int \cos(2x+2)\,dx + \frac{1}{8}\int \cos(4x+4)\,dx$$

$$I = \frac{3}{8}x + \frac{1}{4}\,sen(2x+2) + \frac{1}{32}\,sen(4x+4) + K \quad //$$

(2) Não fazer (c), (d) e (e).

2. (b) $I = \int \frac{1}{\sqrt{9-x^2}}\,dx = \int \frac{3\cos\theta\,d\theta}{\sqrt{9 - 9\,sen^2\theta}} = \frac{\cancel{3}}{\cancel{\sqrt{9}}}\int \frac{\cos\theta\,d\theta}{\sqrt{1-sen^2\theta}}$

$$\boxed{\begin{aligned} x &= 3\,sen\,\theta \\ dx &= 3\cos\theta\,d\theta \end{aligned}} \Rightarrow I = \int \frac{\cos\theta\,d\theta}{\sqrt{\cos^2\theta}} = \int \frac{\cos\theta\,d\theta}{|\cos\theta|} \underset{\cos\theta > 0}{\uparrow}= \int d\theta = \theta + K \quad //$$

$I = \theta = arcsen(x/3)$

FRAÇÕES PARCIAIS

$$\frac{mx+n}{(x-\alpha)(x-\beta)} = \frac{A}{x-\alpha} + \frac{B}{x-\beta} = \frac{A(x-\beta)+B(x-\alpha)}{(x-\alpha)(x-\beta)}$$

$$= \frac{(A+B)x - (A\beta + B\alpha)}{(x-\alpha)(x-\beta)}$$

Ex.1

$$\int \frac{1}{x^2-1}\,dx = \int \frac{dx}{(x-1)(x+1)} = \int dx \left[\frac{A}{x-1} + \frac{B}{x+1}\right]$$

$$= \int dx \left[\frac{(A+B)x + A - B}{(x-1)(x+1)}\right].$$

$$\begin{cases} A+B=0 \Rightarrow A=-B \Rightarrow B=-1/2 \\ A-B=1 \Rightarrow 2A=1 \Rightarrow A=1/2 \end{cases}$$

$$\frac{1}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{1/2}{x-1} - \frac{1/2}{x+1} = \frac{\cancel{1/2\,x} + 1/2 - \cancel{1/2\,x} + 1/2}{(x-1)(x+1)}$$

$A = 1/2$, $B = -1/2$

$$= \frac{1}{x^2-1}.$$

$$\int \frac{dx}{x^2-1} = \int dx \left[\frac{1/2}{x-1} - \frac{1/2}{x+1}\right] = I$$

- -

$$\int \frac{1}{ax+b}\,dx = \int \frac{1}{u}\,\frac{du}{a} = \frac{1}{a}\ln|u| = \frac{1}{a}\ln|ax+b|$$

$u = ax+b$

$du = a\,dx$

- -

$$\Rightarrow I = \frac{1}{2}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{dx}{x+1} = \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + K$$

$$I = \frac{1}{2}\ln\left[\frac{|x-1|}{|x+1|}\right] + K \;//$$

Lista: Sec. 3

3(b) $\displaystyle\int \frac{x}{x^2-4}\,dx = I$

$x^2-4 = (x-2)(x+2)$, $\dfrac{x}{x^2-4} = \dfrac{A}{x-2} + \dfrac{B}{x+2} = \dfrac{Ax+2A+Bx-2B}{(x-2)(x+2)}$

$\Rightarrow \dfrac{x}{x^2-4} = \dfrac{x(A+B)+2A-2B}{(x-2)(x+2)}$

$\Rightarrow \begin{cases} A+B=1 & \Rightarrow 2A=1 \\ 2A-2B=0 & \Rightarrow A=B \end{cases} \Rightarrow A=1/2$

$B=1/2$

$\dfrac{x}{x^2-4} = \dfrac{1/2}{x-2} + \dfrac{1/2}{x+2}$, $I = \displaystyle\int \frac{1/2}{x-2}\,dx + \int \frac{1/2}{x+2}\,dx$

$\Rightarrow I = \dfrac{1}{2}\ln|x-2| + \dfrac{1}{2}\ln|x+2| + k = \dfrac{1}{2}\ln\big[|x-2|\,|x+2|\big] + k$ //

Não fazer (c), (e)

(d) $\displaystyle\int \frac{x+2}{x^2+2x-3}\,dx = I$, $x^2+2x-3=0 \Rightarrow x = \dfrac{1}{2}\left[-2 \pm \sqrt{4+12}\right]$

Raízes: $x_1 = 1$, $x_2 = -3$

$\dfrac{x+2}{x^2+2x-3} = \dfrac{x+2}{(x-1)(x+3)} = \dfrac{A}{x-1} + \dfrac{B}{x+3} = \dfrac{Ax+3A+Bx-B}{(x-1)(x+3)}$

$= \dfrac{(A+B)x+3A-B}{(x-1)(x+3)} \Rightarrow \begin{cases} A+B=1 \\ 3A-B=2 \end{cases}$ ...

(10) $\int \operatorname{arctg}(x)\,dx = x\operatorname{arctg}(x) - \int \frac{x\,dx}{1+x^2} = \dots$

$u = \operatorname{arctg}(x)$; $dv = dx$

$du = \frac{1}{1+x^2}\,dx$; $v = x$

(11) $\int \cos(\ln(x))\,dx = x\cos(\ln x) - \int x\frac{(-1)}{x}\operatorname{sen}(\ln x)\,dx$

$u = \cos(\ln x)$; $dv = dx$; $= x\cos(\ln x) + \underbrace{\int \operatorname{sen}(\ln x)\,dx}$

$du = -\frac{1}{x}\operatorname{sen}(\ln x)\,dx$; $v = x$

Fazer novamente por partes.

SEC. 3

(2) (a) $\int \sqrt{25-4x^2}\,dx = \int \sqrt{4\left[\left(\frac{25}{4}\right) - x^2\right]}\,dx = 2\int \sqrt{\frac{25}{4} - x^2}\,dx = \dots$

$x = a\operatorname{sen}\theta$

$x = \frac{5}{2}\operatorname{sen}\theta \to dx = \frac{5}{2}\cos\theta\,d\theta$