Lista de exercícios 3
Cálculo II

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Nota. *nesta lista, e em todo o material em PDF deste curso, vetores e funções vetoriais são representadas por símbolos em* ***negrito****:* $\boldsymbol{f}(t)$, $\boldsymbol{g}(t)$, $\boldsymbol{x}, \ldots$, *os vetores da base canônica no* $\mathbb{R}^3$ *são* $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ *e a base canônica no* $\mathbb{R}^n$, $n > 3$, *é dada por* $\{\boldsymbol{e}_1, \ \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n\}$

1. Represente, no plano, os pontos com as coordenadas polares dadas. Em seguida, obtenha suas coordenadas cartesianas.

   (a) $(1, \ \pi)$

   (b) $\left(2, \ -\dfrac{2\pi}{3}\right)$

   (c) $\left(-2, \ \dfrac{3\pi}{4}\right)$

   (d) $\left(-\sqrt{2}, \ \dfrac{5\pi}{4}\right)$

   (e) $\left(1, \ \dfrac{5\pi}{2}\right)$

   (f) $\left(2, \dfrac{7\pi}{6}\right)$

2. Dadas as coordenadas cartesianas dos pontos a seguir, obtenha coordenadas polares $(r, \ \theta)$ com $r \geq 0$ e $0 \leq \theta < 2\pi$. Obtenha as coordenadas do mesmo ponto, com $r \geq 0$ e $-\pi \leq \theta < \pi$.

   (a) $(-2, \ 2)$

   (b) $\left(-1, \sqrt{3}\right)$

   (c) $\left(3\sqrt{3}, \ 3\right)$

   (d) $(1, -2)$

   (e) $(0, 4)$

   (f) $\left(\dfrac{1}{2}, \ 0\right)$

3. Esboce a região do plano cujos pontos satisfazem às condições dadas.

   (a) $r \geq 1$

   (b) $0 \leq r < 1$

   (c) $0 \leq r < 2, \qquad \pi \leq \theta \leq \dfrac{3\pi}{2}$

   (d) $1 \leq r \leq 3, \qquad \dfrac{\pi}{6} \leq \theta < \dfrac{5\pi}{6}$

   (e) $r \geq 1, \qquad \pi \leq \theta \leq 2\pi$

   (f) $2 < r \leq 3, \qquad \dfrac{5\pi}{3} \leq \theta < \dfrac{7\pi}{3}$

4. Esboce a curva dada em coordenadas polares:

   (a) $r = e^{-\theta}, \qquad \theta \geq 0$

   (b) $r = \cos\theta$

   (c) $r\cos\theta = 1, \qquad -\dfrac{\pi}{2} < \theta < \dfrac{pi}{2}$

   (d) $r = 2$

   (e) $\theta = \dfrac{\pi}{3}$

   (f) $r = \cos 4\theta$

5. Passe a curva dada para coordenadas polares e a desenhe

(a) $x^4 - y^4 = 2xy$

(b) $x^2 + y^2 + x = \sqrt{x^2 + y^2}$

(c) $\left(x^2 + y^2\right)^{\frac{3}{2}} = y^2$

(d) $\left(x^2 + y^2\right)^2 = x^2 - y^2$

6. Calcule a área da região limitada pela curva dada
   (a) $r = 2 - \cos\theta$
   (b) $r = \cos 2\theta$
   (c) $r = \cos 3\theta$

7. Determine o domínio das seguintes funções $\boldsymbol{f} : \mathbb{R} \longrightarrow \mathbb{R}^n$:
   (a) $\boldsymbol{f}(t) = \left(\sqrt{4 - t^2}, e^{-3t}, \ln(t+1)\right)$
   (b) $\boldsymbol{f}(t) = \left(t, \sqrt{\dfrac{t-2}{t+1}}, \ln(5 - t^2), e^{-t}\right)$

8. Desenhe a imagem das seguintes funções $\boldsymbol{f} : \mathbb{R} \longrightarrow \mathbb{R}^2$
   (a) $\boldsymbol{f}(t) = (2t - 1, t + 2)$
   (b) $\boldsymbol{f}(t) = (\cos t, \operatorname{sen} t)$
   (c) $\boldsymbol{f}(t) = (\cos t, 2\operatorname{sen} t)$
   (d) $\boldsymbol{f}(t) = (\operatorname{sen} t, \operatorname{sen} t)$
   (e) $\boldsymbol{f}(t) = (e^t \cos t, e^t \operatorname{sen} t),\ t \geq 0$

9. Desenhe a imagem das seguintes funções $\boldsymbol{f} : \mathbb{R} \longrightarrow \mathbb{R}^3$
   (a) $\boldsymbol{f}(t) = (t, t, t^2),\ t \geq 0$
   (b) $\boldsymbol{f}(t) = (t, t, 1 + \operatorname{sen} t),\ t \geq 0$
   (c) $\boldsymbol{f}(t) = (\cos t, \operatorname{sen} t, 5)$
   (d) $\boldsymbol{f}(t) = (\cos t, \operatorname{sen} t, t),\ t \geq 0$

10. Dadas $\boldsymbol{f}(t) = (t, \operatorname{sen} t, 2)$ e $\boldsymbol{g}(t) = \left(3, t, t^2\right)$, calcule:
    (a) $(\boldsymbol{f} + \boldsymbol{g})(t)$
    (b) $(\boldsymbol{f} - \sqrt{2}\boldsymbol{g})(t)$
    (c) $(\boldsymbol{f} \cdot \boldsymbol{g})(t)$
    (d) $(\boldsymbol{f} \times \boldsymbol{g})(t)$
    (e) $e^t \boldsymbol{f}(t)$

11. Calcule $\dfrac{\mathrm{d}\boldsymbol{f}}{\mathrm{d}t}$ e $\dfrac{\mathrm{d}^2\boldsymbol{f}}{\mathrm{d}t^2}$:
    (a) $\boldsymbol{f}(t) = (\cos t, \operatorname{sen} t, t)$
    (b) $\boldsymbol{f}(t) = \left(3t^2, e^{-t}, \ln(t^2 + 1)\right)$
    (c) $\sqrt[3]{t^2}\boldsymbol{i} + \cos t^2 \boldsymbol{j} + 3t\boldsymbol{k}$
    (d) $\left(\tan t, \sec t, \dfrac{1}{t^2}\right)$

12. Determine o vetor unitário tangente $\boldsymbol{T}(t)$ no ponto determinado pelo parâmetro t, dado:
    (a) $\boldsymbol{f}(t) = \left(te^{-t}, 3\arctan t, 2e^t\right),\ t = 0$
    (b) $\boldsymbol{f}(t) = \left(t^3, 3t, t^2 + 1, 3t + 4\right),\ t = 1$
    (c) $\boldsymbol{f}(t) = \operatorname{sen}^2 t\boldsymbol{i} + \cos^2 t\boldsymbol{j} + \tan^2 t\boldsymbol{k},\ t = \dfrac{\pi}{4}$

13. Demonstre as seguintes propriedades:

(a) $\dfrac{\mathrm{d}}{\mathrm{d}t}\left[\boldsymbol{f}(t)+\boldsymbol{g}(t)\right]=\dfrac{\mathrm{d}\boldsymbol{f}}{\mathrm{d}t}+\dfrac{\mathrm{d}\boldsymbol{g}}{\mathrm{d}t}$

(b) $\dfrac{\mathrm{d}}{\mathrm{d}t}\left[h(t)\boldsymbol{f}(t)\right]=\dfrac{\mathrm{d}h}{\mathrm{d}t}\boldsymbol{f}(t)+h(t)\dfrac{\mathrm{d}\boldsymbol{f}}{\mathrm{d}t}$, onde $h(t)$ é uma função escalar

(c) $\dfrac{\mathrm{d}}{\mathrm{d}t}\left[\boldsymbol{f}(t)\cdot\boldsymbol{g}(t)\right]=\dfrac{\mathrm{d}\boldsymbol{f}}{\mathrm{d}t}\cdot\boldsymbol{g}(t)+\boldsymbol{f}(t)\cdot\dfrac{\mathrm{d}\boldsymbol{g}}{\mathrm{d}t}$

(d) $\dfrac{\mathrm{d}}{\mathrm{d}t}\left[\boldsymbol{f}(t)\times\boldsymbol{g}(t)\right]=\dfrac{\mathrm{d}\boldsymbol{f}}{\mathrm{d}t}\times\boldsymbol{g}(t)+\boldsymbol{f}(t)\times\dfrac{\mathrm{d}\boldsymbol{g}}{\mathrm{d}t}$

14. Seja $\boldsymbol{f}: A \longrightarrow \mathbb{R}^n$ derivável tal que $\|\boldsymbol{f}(t)\| = k$, k constante, para todo $t \in A$. Prove que $\boldsymbol{f}(t)\cdot\dfrac{\mathrm{d}\boldsymbol{f}}{\mathrm{d}t}=0$ para todo $t \in A$. Interprete geometricamente no caso $n = 2$.

15. Obtenha o comprimento das seguintes curvas:

    (a) $\boldsymbol{f}(t) = (t,\ 3\cos t,\ 3\,\mathrm{sen}\,t)$, $\qquad -5 \le t \le 5$

    (b) $\boldsymbol{r}(t) = \left(2t,\ t^2,\ \dfrac{1}{3}t^3\right)$, $\qquad 0 \le t \le 1$

    (c) $\boldsymbol{f}(t) = \sqrt{2}t\boldsymbol{i} + e^t\boldsymbol{j} + e^{-t}\boldsymbol{k}$, $\qquad 0 \le t \le 1$

    (d) $\boldsymbol{f}(t) = \cos(t)\boldsymbol{i} + \mathrm{sen}\,(t)\boldsymbol{j} + \ln(\cos t)\boldsymbol{k}$, $\qquad 0 \le t \le \dfrac{\pi}{4}$

16. Obtenha os vetores unitários tangente ($\boldsymbol{T}(t)$) e normal ($\boldsymbol{N}(t)$). Obtenha a curvatura $\kappa(t)=\dfrac{\|\boldsymbol{T}'(t)\|}{\|\boldsymbol{r}'(t)\|}$

    (a) $\boldsymbol{r}(t) = (t,\ 3\cos t,\ 3\,\mathrm{sen}\,t)$

    (b) $\boldsymbol{r}(t) = \left(\sqrt{2}t,\ e^t,\ e^{-t}\right)$

17. Em que ponto da curva $\gamma(t) = \left(t^3,\ 3t,\ t^4\right)$ o vetor tangente é perpendicular ao plano $6x + 6y - 8z = 1$?