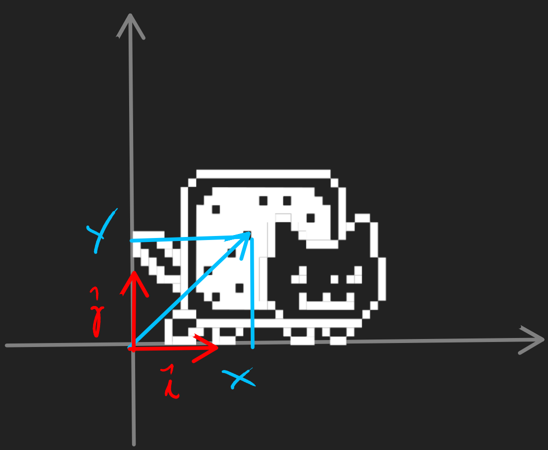
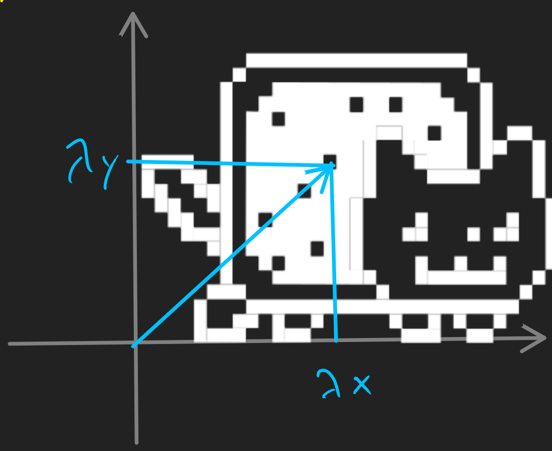
Notas de aula ECT2202 T02 2022-01-13 Aula 17 — Transf. Lineares e Geometria

Transformação de escala (ampliação/redução)





λ fator de escala

$\lambda > 0$

$S_\lambda(x,y) = (\lambda x, \lambda y)$

$0 < \lambda < 1$ redução

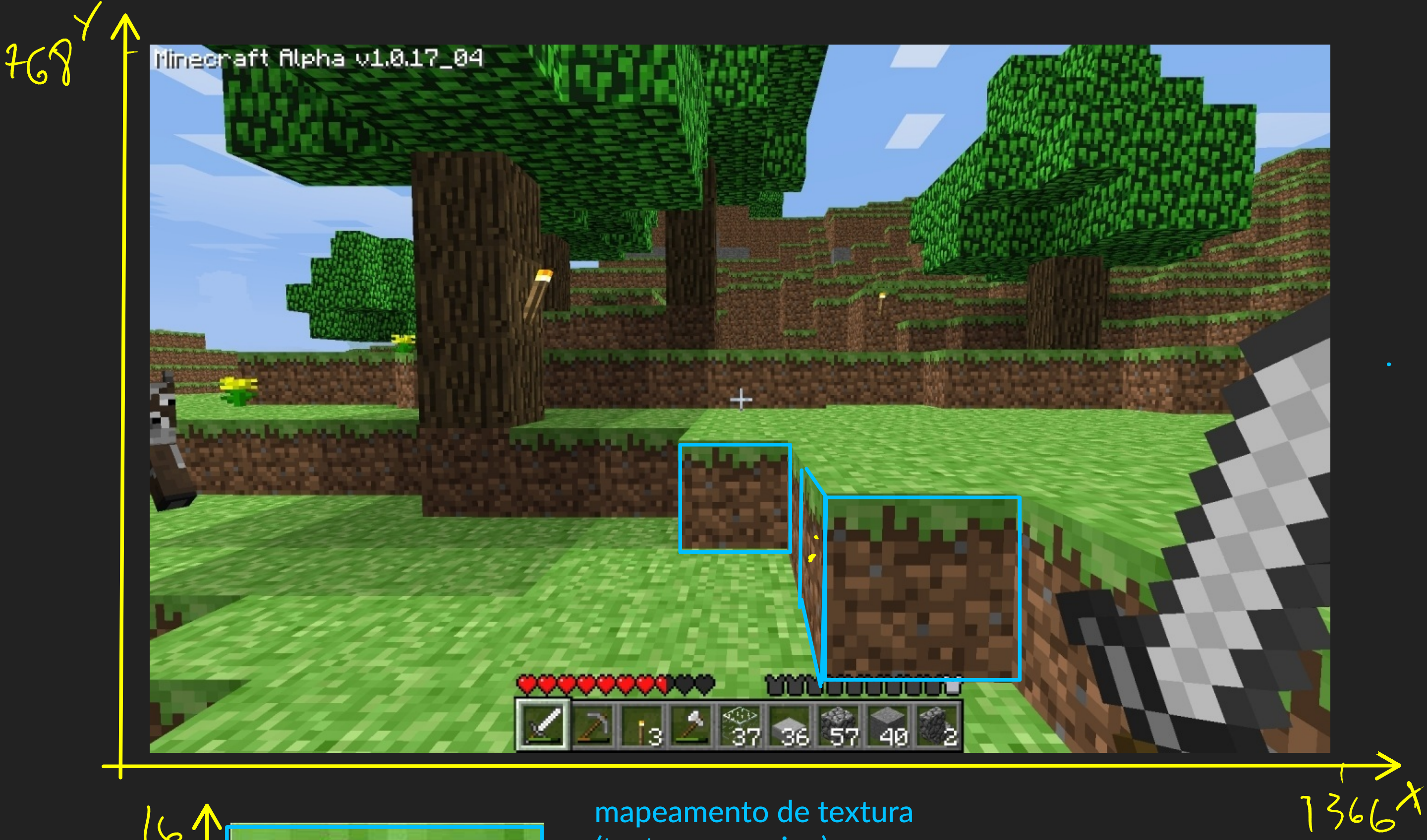
$\lambda > 1$ ampliação

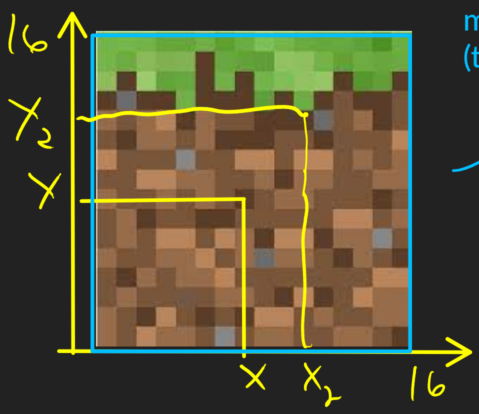
$$[S_\lambda] = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$[S_\lambda(x,y)] = [S_\lambda]\begin{bmatrix} x \\ y \end{bmatrix}$$

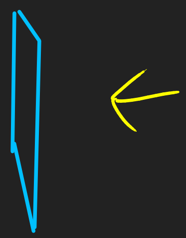
Em dimensão 3

$$[S_\lambda] = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$





mapeamento de textura
(texture mapping)



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = [T] \begin{bmatrix} x \\ y \end{bmatrix}$$

Escala assimétrica

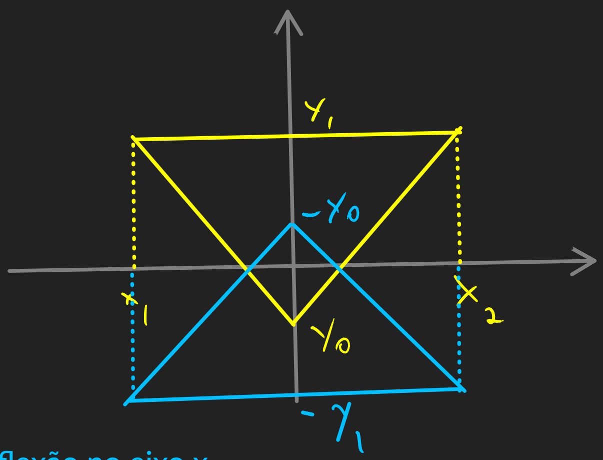
$$S_x(x, y) = (\lambda x, y)$$

$$[S_x] = \begin{bmatrix} \lambda & 0 \\ 0 & 1 \end{bmatrix}$$

$$S_{xy}(x, y) = (\lambda_1 x, \lambda_2 y) \Rightarrow [S_{x,y}] = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$[S_{xy}] = [S_x][S_y]$$ (verifique)

Reflexões



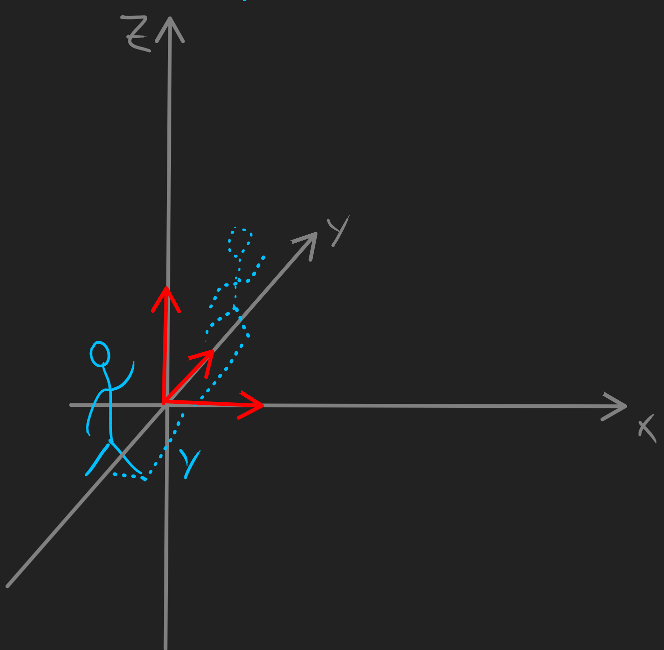
$$(x_1, y_1) \to (x_1, -y_1)$$
$$(x_2, y_1) \to (x_2, -y_1)$$
$$(0, y_0) \to (0, -y_0)$$

Reflexão no eixo x

$$E_x(x, y) = (x, -y)$$

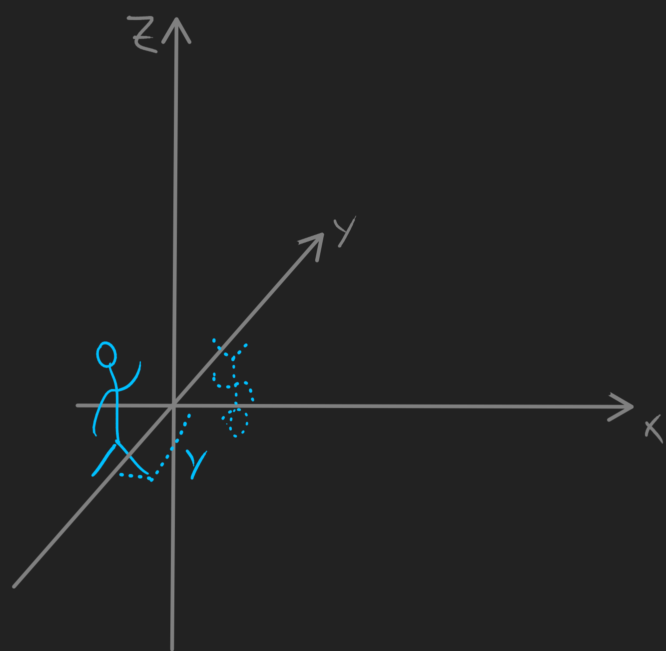
$$[E_x] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Em 3 dimensões:
Reflexão no plano xz:



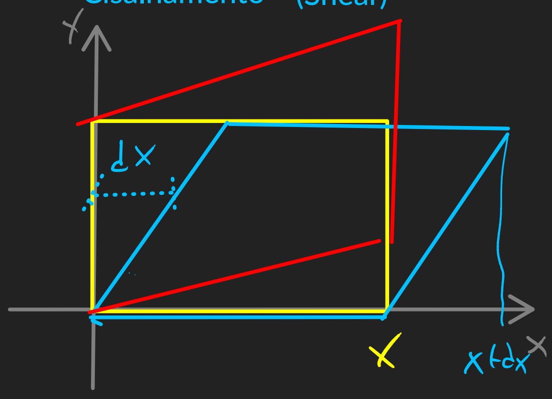
$$[E_{xz}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Reflexão (inversão) em relação à origem



$$[E_{xyz}] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

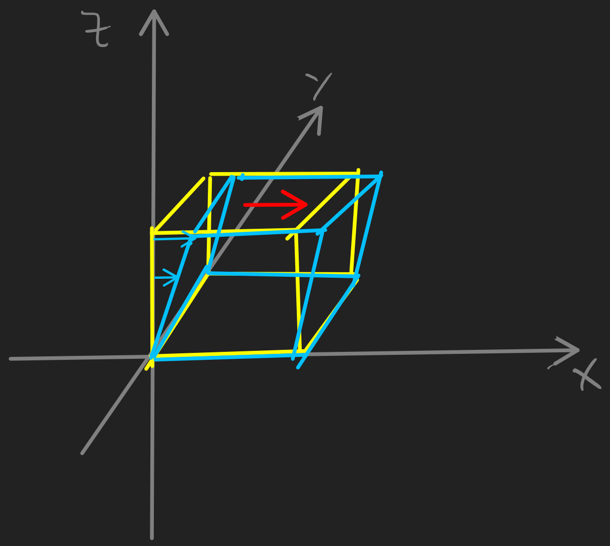
Cisalhamento (Shear)



$$dx = \alpha y$$

$$C_x(x,y) = (x+\alpha y,\ y)$$

$$[C_x] = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}$$

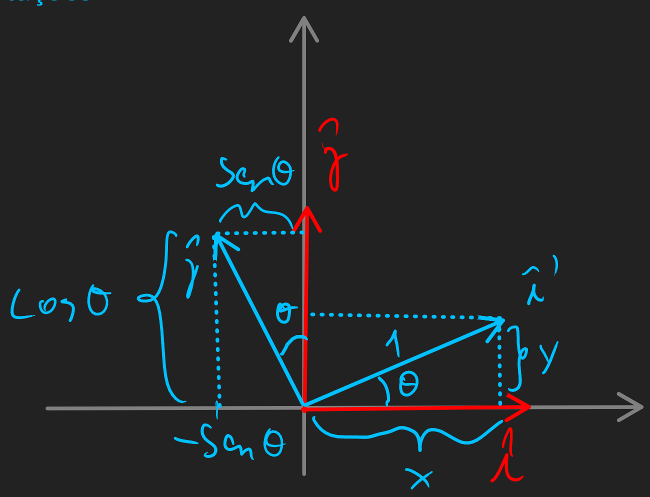


Cisalhamento na direção x em função da coordenada z

$$C_{x,z}(x,y,z) = (x+\alpha z,\ y,\ z)$$

$$[C_{x,z}] = \begin{bmatrix} 1 & 0 & \alpha \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotações



$$\hat{i}' = (\cos\theta,\ \mathrm{sen}\,\theta)$$

$$\hat{j}' = (-\mathrm{sen}\,\theta,\ \cos\theta)$$

$$\cos\theta = \frac{CA}{H}$$

$$\mathrm{sen}\,\theta = \frac{CO}{H}$$

$$[R_\theta] = \begin{bmatrix} \cos\theta & -\mathrm{sen}\,\theta \\ \mathrm{sen}\,\theta & \cos\theta \end{bmatrix}$$

$$\det[R_\theta] = 1$$

$$[R_{(-\theta)}] = \begin{bmatrix} \cos(-\theta) & -\mathrm{sen}(-\theta) \\ \mathrm{sen}(-\theta) & \cos(-\theta) \end{bmatrix} = \begin{bmatrix} \cos\theta & \mathrm{sen}\,\theta \\ -\mathrm{sen}\,\theta & \cos\theta \end{bmatrix}$$

$$[R_{(-\theta)}] = [R_\theta]^T \quad \Rightarrow \quad [R_\theta]^{-1} = [R_\theta]^T$$

$$[R_{(-\theta)}] = [R_\theta]^{-1} \leftarrow$$

Matriz ortogonal
unitária

$$[R_{(-\theta)}][R_\theta] = \mathbb{1}$$