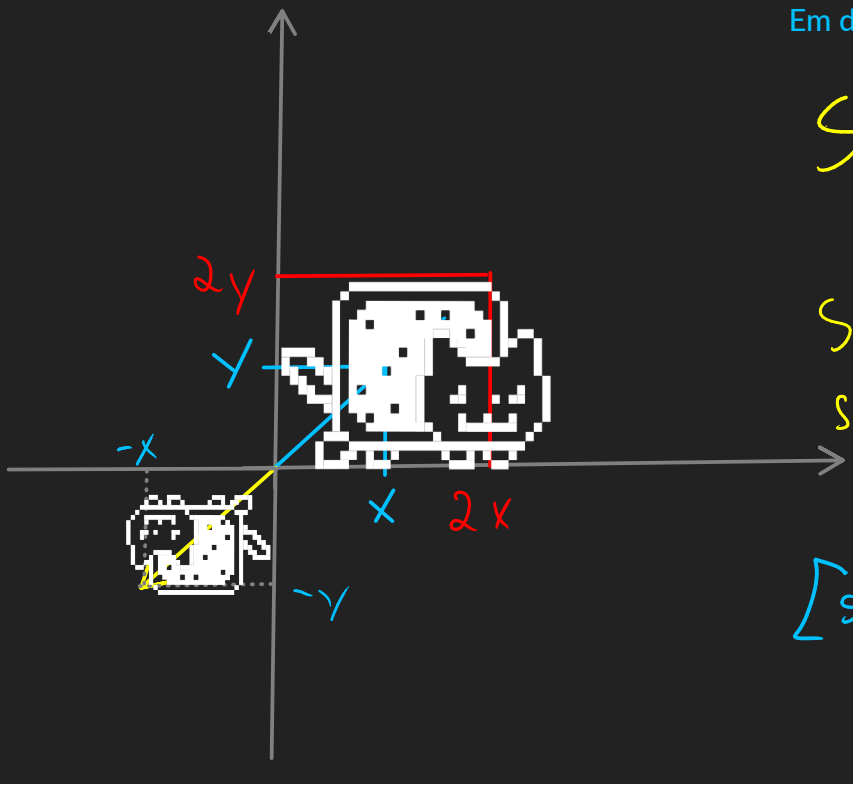
Notas de aula ECT2202 T03 2022-01-13 Aula 17 — Tranf. Lineares e Geometria

Textura

16x16 pixels

Escala (ampliação/redução)



Em dimensão 2

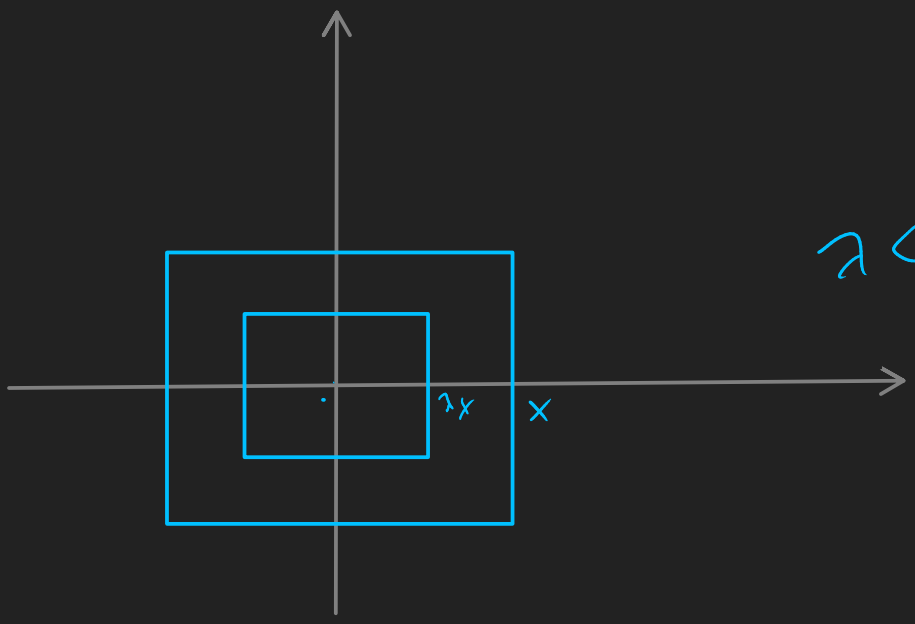
$$S(x,y) = (\lambda x, \lambda y)$$

$\lambda > 0$

Se $0 < \lambda < 1$ redução

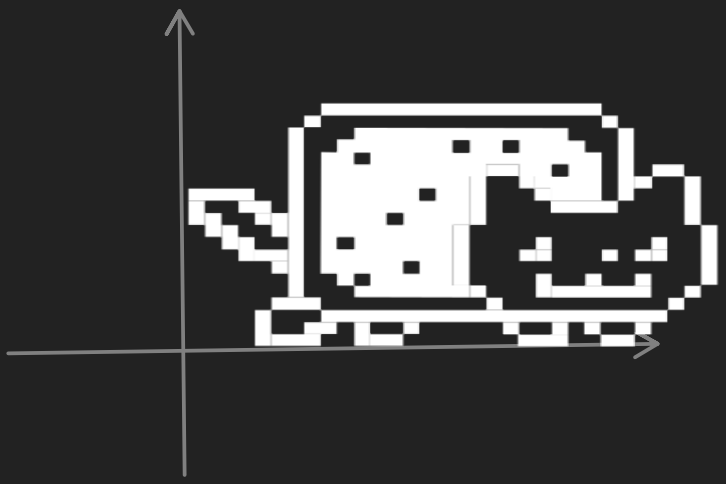
Se $\lambda > 1$ ampliação

$$[S] = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$



$\lambda < 1$

Escala assimétrica



$S_x(x,y) = (\lambda x, y)$

$\lambda > 0$

$0 < \lambda < 1$. redução

$\lambda > 1$ ampliação

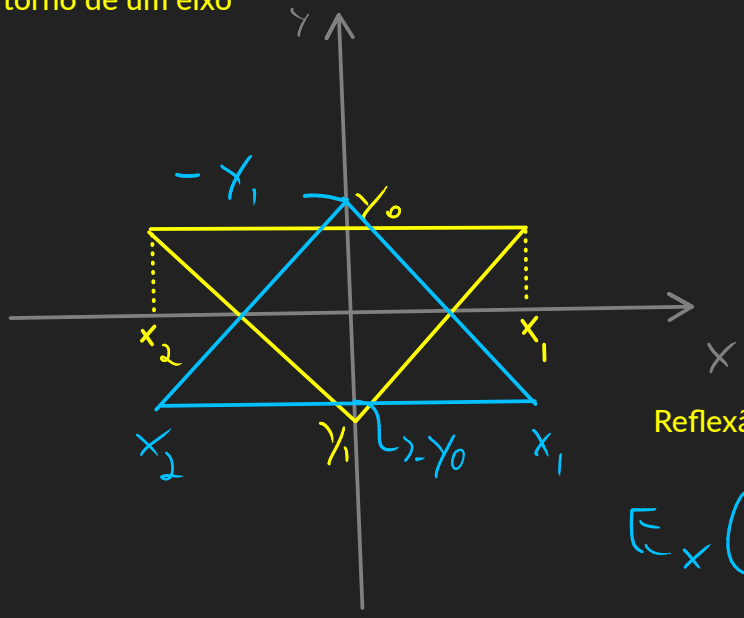
$$[S] = \begin{bmatrix} \lambda & 0 \\ 0 & 1 \end{bmatrix}$$

$$S_{xy}(x,y) = (\lambda_1 x, \lambda_2 y)$$

$$S_{xy} = S_x \circ S_y = S_y \circ S_x$$

$$[S_{xy}] = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Reflexão em torno de um eixo



$(x_1, y_0) \rightarrow (x_1, -y_0)$

$(x_2, y_0) \rightarrow (x_2, -y_0)$

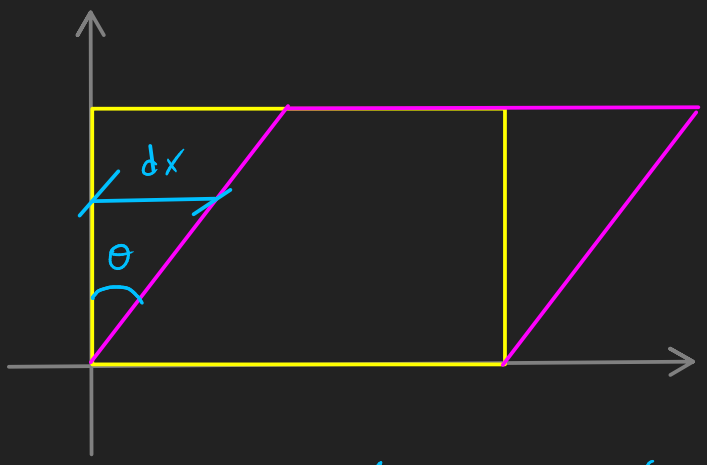
$(0, y_1) \rightarrow (0, -y_1)$

Reflexão em torno do eixo x

$$E_x(x, y) = (x, -y)$$

$$[E_x] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Cisalhamento (Shear)



$$dx = \alpha y$$

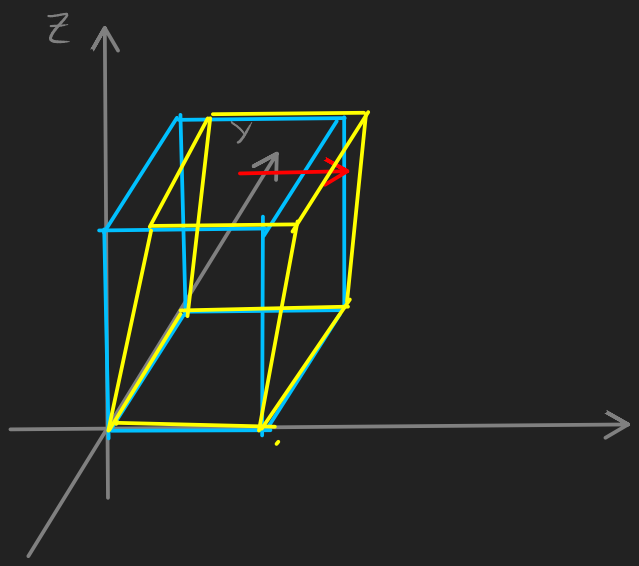
θ ângulo de cisalhamento

$$\tan(\theta) = \alpha$$

$$C_x(x, y) = (x + \alpha y, y)$$

$$[C_x] = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}$$

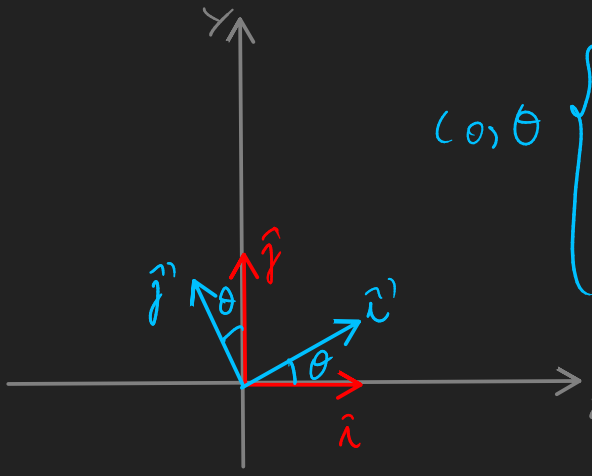
Cisalhamento na direção X em função de z

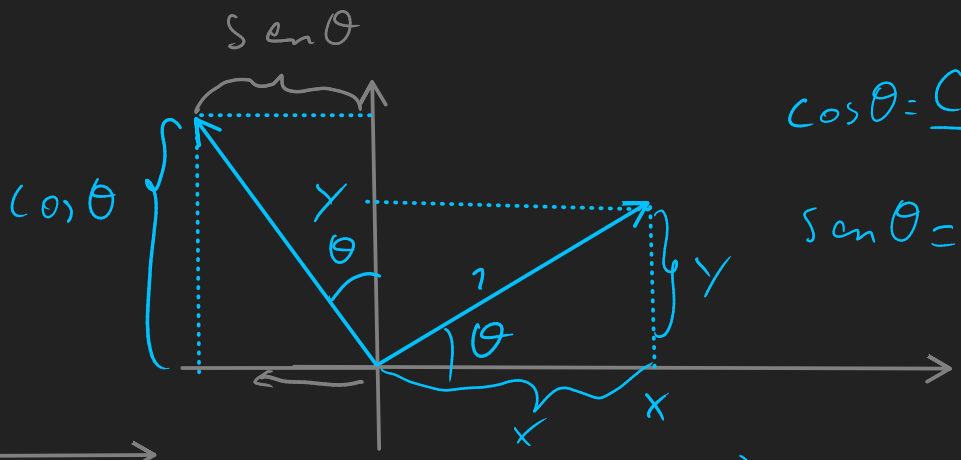


$$[C_{x,z}] = \begin{bmatrix} 1 & 0 & \alpha \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_{x,z}(x, y, z) = (x + \alpha z, y, z)$$

Rotação no plano





$$\cos\theta = \frac{CA}{H}$$

$$\operatorname{sen}\theta = \frac{CO}{H}$$

$$\hat{i}' = (\cos\theta, \operatorname{sen}\theta)$$

$$\hat{j}' = (-\operatorname{sen}\theta, \cos\theta)$$

$$R_\theta(1,0) = (\cos\theta, \operatorname{sen}\theta)$$

$$R_\theta(0,1) = (-\operatorname{sen}\theta, \cos\theta)$$

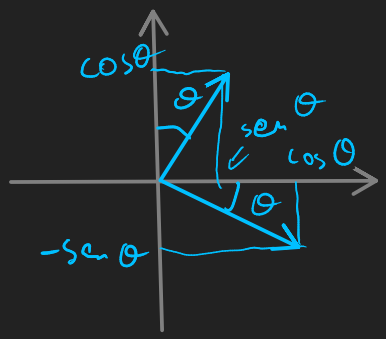
$$[R_\theta] = \begin{bmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{bmatrix}$$

$\theta > 0$

$\theta < 0$

$$R_{(-\theta)} = \begin{bmatrix} \cos(-\theta) & -\operatorname{sen}(-\theta) \\ \operatorname{sen}(-\theta) & \cos(-\theta) \end{bmatrix}$$

$$\cos(-\theta) = \cos\theta$$
$$\operatorname{sen}(-\theta) = -\operatorname{sen}\theta$$



$$R_{(-\theta)} = \begin{bmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{bmatrix}$$

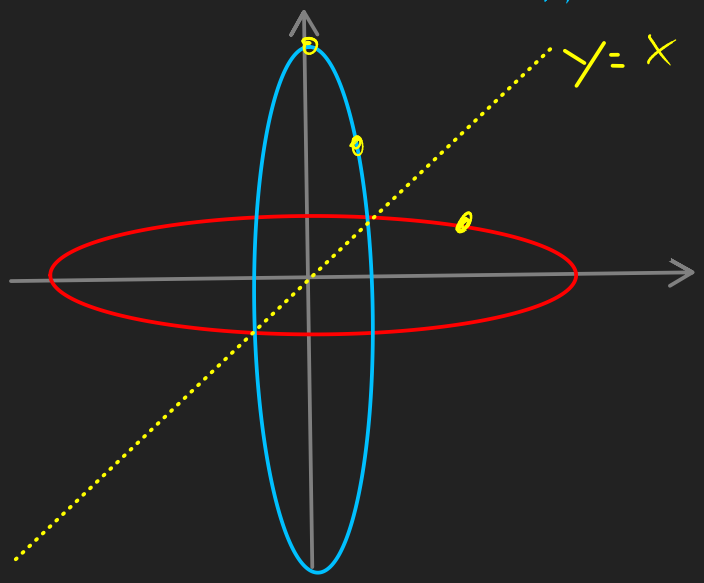
$$(R_\theta)^{-1} = R_{(-\theta)}$$

$$\begin{bmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{bmatrix}^{-1} = \begin{bmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{bmatrix}$$

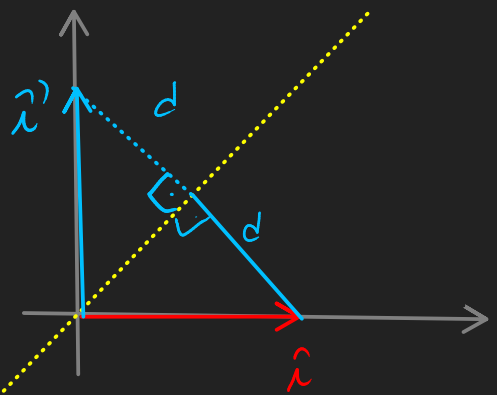
$$[R_\theta]^{-1} = [R_\theta]^T$$

matriz ortogonal

matriz unitária

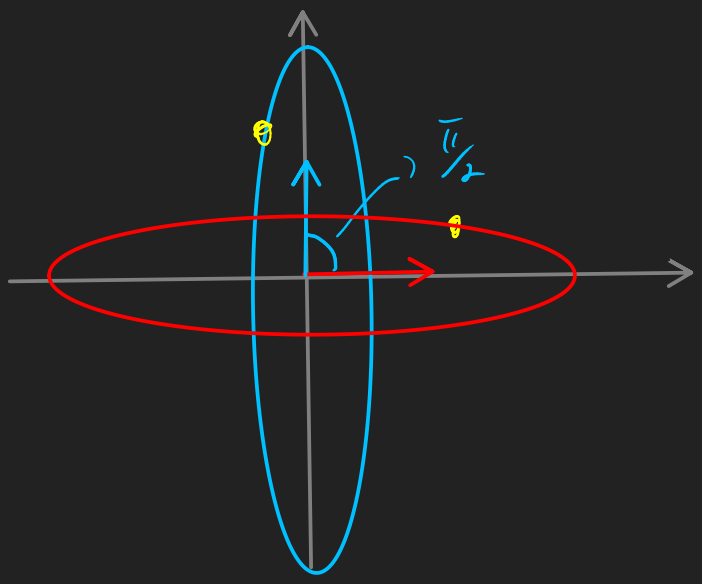


Reflexão em torno de reta $y = x$



$$E_{xy}(x, y) = (y, x)$$

$$[E_{xy}] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow \det[E_{xy}] = -1$$



$$\left[R_{\frac{\pi}{2}}\right] = \begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$