13. Mostre que conjunto $\boxed{B = \{(1,1,0),(1,-1,1),(1,0,-1)\} \text{ é uma base do } \mathbb{R}^3.}$

14. Obtenha $[I]^B_C$, a matriz mudança de base da base $B$ do exercício anterior para a base canônica $C = \{(1,0,0),(0,1,0),(0,0,1)\}$ de $\mathbb{R}^3$. Obtenha em seguida $[I]^C_B$. Mostre que $\boxed{[I]^B_C = \left([I]^C_B\right)^{-1}.}$

$$B = \{\underset{u_1}{(1,1,0)}, \underset{u_2}{(1,-1,1)}, \underset{u_3}{(1,0,-1)}\}$$

$$C = \{\underset{e_1}{(1,0,0)}, \underset{e_2}{(0,1,0)}, \underset{e_3}{(0,0,1)}\}$$

$$v = (a,b,c) = \alpha u_1 + \beta u_2 + \gamma u_3$$

$$[v]_B = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}_B \qquad v = a e_1 + b e_2 + c e_3 \qquad [v]_C = \begin{bmatrix} a \\ b \\ c \end{bmatrix}_C$$

Sei $[v]_B$ e quero $[v]_C$

$$[v]_C = [I]^B_C [v]_B \qquad B = \{v_1, v_2, v_3\}$$

$$[I]^B_C = \begin{bmatrix} [v_1]_C & [v_2]_C & [v_3]_C \end{bmatrix}$$

$$[v_1]_C = [(1,1,0)]_C = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad v_1 = 1(1,0,0) + 1(0,1,0) + 0(0,0,1)$$

$$[v_2]_C = [(1,-1,1)]_C = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \qquad [I]^B_C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

$$[v_3]_C = [(1,0,-1)]_C = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$[I]^C_B = \begin{bmatrix} [(1,0,0)]_B & [(0,1,0)]_B & [(0,0,1)]_B \end{bmatrix}$$

$$B = \{\underset{v_1}{(1,1,0)}, \underset{v_2}{(1,-1,1)}, \underset{v_3}{(1,0,-1)}\}$$

$$[(1,0,0)]_B = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$(1,0,0) = \alpha v_1 + \beta v_2 + \gamma v_3$$
$$= \alpha(1,1,0) + \beta(1,-1,1) + \gamma(1,0,-1)$$

$$(1,0,0) = (\alpha+\beta+\gamma, \ \alpha-\beta, \ \beta-\gamma)$$ equação vetorial

$$\begin{cases} \alpha+\beta+\gamma = 1 \\ \alpha-\beta = 0 \\ \beta-\gamma = 0 \end{cases} \Rightarrow \begin{cases} \alpha+\alpha+\alpha = 1 \Rightarrow 3\alpha = 1 \\ \alpha = \beta \\ \beta = \gamma \end{cases}$$

$$\alpha = 1/3 \quad \beta = 1/3 \quad \gamma = 1/3$$

$$[(1,0,0)]_B = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} \checkmark$$

$$[(0,1,0)]_B = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 2/3 \\ -1/3 \\ -1/3 \end{bmatrix}$$

$$(0,1,0) = (\alpha+\beta+\gamma, \ \alpha-\beta, \ \beta-\gamma)$$ equação vetorial

$$\begin{cases} \alpha+\beta+\gamma = 0 \\ \alpha-\beta = 1 \\ \beta-\gamma = 0 \end{cases} \Rightarrow \begin{cases} (\beta+1)+\beta+\beta = 0 \Rightarrow 3\beta = -1 \\ \alpha = \beta+1 \\ \beta = \gamma \end{cases}$$

$$\beta = -1/3 \quad \gamma = -1/3 \quad \alpha = 2/3$$

$$[(0,0,1)]_B = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ -2/3 \end{bmatrix}$$

$$(0,0,1) = (\alpha+\beta+\gamma,\ \alpha-\beta,\ \beta-\gamma)$$ equação vetorial

$$\begin{cases} \alpha+\beta+\gamma = 0 \\ \alpha-\beta = 0 \\ \beta-\gamma = 1 \end{cases} \Rightarrow \begin{cases} \beta+\beta+(\beta-1)=0 \\ \alpha=\beta \\ \gamma=\beta-1 \end{cases} \Rightarrow 3\beta=1$$

$$\beta = 1/3 \quad \alpha = 1/3 \quad \gamma = -2/3$$

$$[I]^C_B = \begin{bmatrix} 1/3 & 2/3 & 1/3 \\ 1/3 & -1/3 & 1/3 \\ 1/3 & -1/3 & -2/3 \end{bmatrix} \checkmark$$

$$[(1,-1,1)]_C = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}_C$$

$$[(1,-1,1)]_B = [I]^C_B\,[(1,-1,1)]_C$$

$$= \begin{bmatrix} 1/3 & 2/3 & 1/3 \\ 1/3 & -1/3 & 1/3 \\ 1/3 & -1/3 & -2/3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}_C = \begin{bmatrix} 1/3 - 2/3 + 1/3 \\ 1/3 - (-1/3) + 1/3 \\ 1/3 - (-1/3) + (-2/3) \end{bmatrix}$$

$$[(1,-1,1)]_B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}_B \Rightarrow (1,-1,1) = 0\,(1,1,0) + 1\,(1,-1,1) + 0\,(1,0,-1)$$

$$[I]^B_C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \qquad [I]^C_B = \begin{bmatrix} 1/3 & 2/3 & 1/3 \\ 1/3 & -1/3 & 1/3 \\ 1/3 & -1/3 & -2/3 \end{bmatrix}$$

Mostrar que $[I]^B_C = \left([I]^C_B\right)^{-1}$

$$[I]^C_B \left([I]^C_B\right)^{-1} = \mathbb{1}$$

$$[I]^C_B \, [I]^B_C = \mathbb{1}$$

$$\begin{bmatrix} 1/3 & 2/3 & 1/3 \\ 1/3 & -1/3 & 1/3 \\ 1/3 & -1/3 & -2/3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1/3 + 2/3 & \frac{1}{3} - 2/3 + 1/3 & \frac{1}{3} - \frac{1}{3} \\ 1/3 - 1/3 & 1/3 - \left(-\frac{1}{3}\right) + \frac{1}{3} & \frac{1}{3} - \frac{1}{3} \\ 1/3 - 1/3 & \frac{1}{3} - \left(-\frac{1}{3}\right) - \frac{2}{3} & \frac{1}{3} - \left(\frac{-2}{3}\right) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

18. Mostre que o conjunto $\{1, \text{sen}(t), \text{sen}(2t)\}$ de vetores de $C(\mathbb{R})$ é linearmente independente.

espaço vetorial das funções reais contínuas.

L.I.:

$0 \in C(\mathbb{R})$

$$(I) \quad \alpha(1) + \beta \,\text{sen}(t) + \gamma \,\text{sen}(2t) = 0 \implies \alpha = \beta = \gamma = 0$$

1 equação vetorial
3 variáveis.

para qualquer valor de t.

Obter 3 equações: uma para cada valor de t.

$t = 0$, $t = \pi/2$, $t = \pi/4$

$t = 0$

$$\alpha(1) + \beta \,\text{sen}(0) + \gamma \,\text{sen}(0) = 0 \quad (1)$$

$$\alpha + 0\beta + 0\gamma = 0 \implies \alpha = 0$$

$t = \pi/2$

$$\alpha(1) + \beta \,\text{sen}(\pi/2) + \gamma \,\text{sen}(2\,\pi/2) = 0$$

$$0 + \beta + 0\gamma = 0 \implies \beta = 0$$

$t = \pi/4$

$$\alpha(1) + \beta \,\text{sen}(\pi/4) + \gamma \,\text{sen}(2\,\pi/4) = 0$$

$$0(1) + 0 \,\text{sen}(\pi/4) + \gamma \,\text{sen}(\pi/2) = 0$$

$$\implies \gamma = 0$$

Logo o conj é L.I.

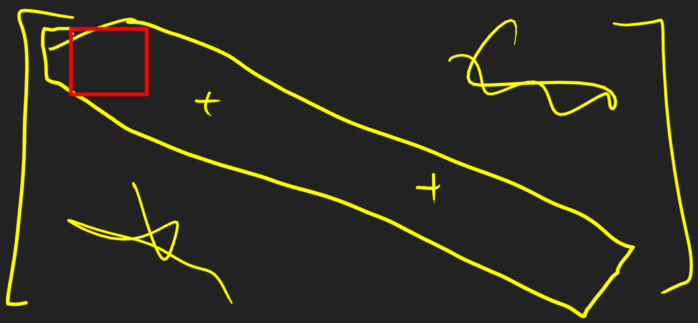
Produto interno

Espaço das matrizes

$$A = \begin{bmatrix} \boxed{\begin{matrix} a_{11} \\ \vdots \\ a_{n1} \end{matrix}} & \cdots & a_{1n} \\ & & \\ & \cdots & a_{nn} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & & \\ b_{n1} & \cdots & b_{nn} \end{bmatrix}$$

$$B^t = \begin{bmatrix} \boxed{b_{11} \quad \cdots \quad b_{n1}} \\ \vdots \\ b_{1n} \quad \cdots \quad b_{nn} \end{bmatrix}$$

$B^t A =$




$$\boxed{tr\left(B^t A\right)} =$$