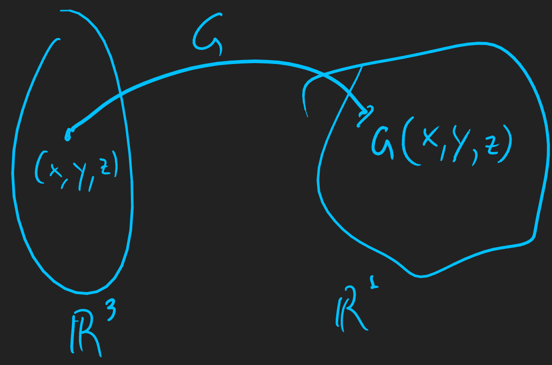
Notas de aula ECT2202 — 2021-07-21 — Aula 12 — Transformações Lineares

$G: \mathbb{R}^3 \to \mathbb{R}^2$ $\boxed{G(x, y, z) = (x + 2y - 4z, 2x + 3y + z)}$



a) G é linear?

$$\begin{cases} G(\overbrace{u+v}^{\mathbb{R}^3}) = \overbrace{G(u)+G(v)}^{\mathbb{R}^2} \\ G(\lambda u) = \lambda G(u) \end{cases}$$

Sejam $u = (x, y, z) \in \mathbb{R}^3$

$v = (a, b, c) \in \mathbb{R}^3$

$\lambda \in \mathbb{R}$

$G(x, y, z) = (x + 2y - 4z, 2x + 3y + z)$

$$G(u+v) = G((x,y,z) + (a,b,c))$$

$$= G(x+a, y+b, z+c)$$

$$= ((x+a) + 2(y+b) - 4(z+c), 2(x+a) + 3(y+b) + (z+c))$$

$$= ((x+2y-4z) + (a+2b-4c), (2x+3y+z) + (2a+3b+c))$$

$$= \underbrace{(x+2y-4z, 2x+3y+z)}_{G(x,y,z)} + \underbrace{(a+2b-4c, 2a+3b+c)}_{G(a,b,c)} \quad //$$

$$G(\lambda u) = G(\lambda(x,y,z)) = G(\lambda x, \lambda y, \lambda z)$$

$$= (\lambda x + 2\lambda y - 4\lambda z, 2\lambda x + 3\lambda y + \lambda z)$$

$$= (\lambda(x+2y-4z), \lambda(2x+3y+z))$$

$$= \lambda(x+2y-4z, 2x+3y+z) = \lambda G(x,y,z) \quad //$$

Logo G é Linear //

$G(x, y, z) = (x + 2y - 4z, 2x + 3y + z)$

b) Calcule $G(1, -5, 3)$

$$G(1,-5,3) = (1 + 2(-5) - 4(3), 2(1) + 3(-5) + 3)$$
$$= (1 - 10 - 12, 2 - 15 + 3)$$
$$= (-21, -10)$$

c) Calcule $\underbrace{G(0,0,0)}_{\emptyset_{\mathbb{R}^3}} = \emptyset_{\mathbb{R}^2} = (0,0)$

$$G(0,0,0) = (0 + 2(0) - 4(0), 2(0) + 3(0) + 0) = (0,0) \checkmark$$

d) $B = \{(1,0,0), (0,1,0), (0,0,1)\}$ base de $\mathbb{R}^3$

$$G(B) = \{G(1,0,0), G(0,1,0), G(0,0,1)\}$$

$G(x, y, z) = (x + 2y - 4z, 2x + 3y + z)$

$$G(1,0,0) = (1 + 2(0) - 4(0), 2(1) + 3(0) + 0)$$
$$= (1, 2)$$
$$G(0,1,0) = (2, 3)$$
$$G(0,0,1) = (-4, 1)$$

$$G(B) = \{(1,2), (2,3), (-4,1)\}$$

ELEMENTOS DE MATRIZ
DA TRANSFORMAÇÃO G

$$\begin{cases} G(1,0,0) = (1,2 \\ G(0,1,0) = (2,3) \\ G(0,0,1) = (-4,1) \end{cases}$$

$$G(a,b,c) = G(a(1,0,0) + b(0,1,0) + c(0,0,1))$$

Como G é linear:

$$= a\,G(1,0,0) + b\,G(0,1,0) + c\,G(0,0,1)$$
$$= a(1,2) + b(2,3) + c(-4,1)$$
$$= (a + 2b - 4c,\ 2a + 3b + c)$$

$$G(x, y, z) = (x + 2y - 4z, 2x + 3y + z)$$

$$G(2,-5,4) = 2(1,2) - 5(2,3) + 4(-4,1)$$
$$= (2,4) + (-10,-15) + (-16,4)$$
$$= (-24,-7)$$ //

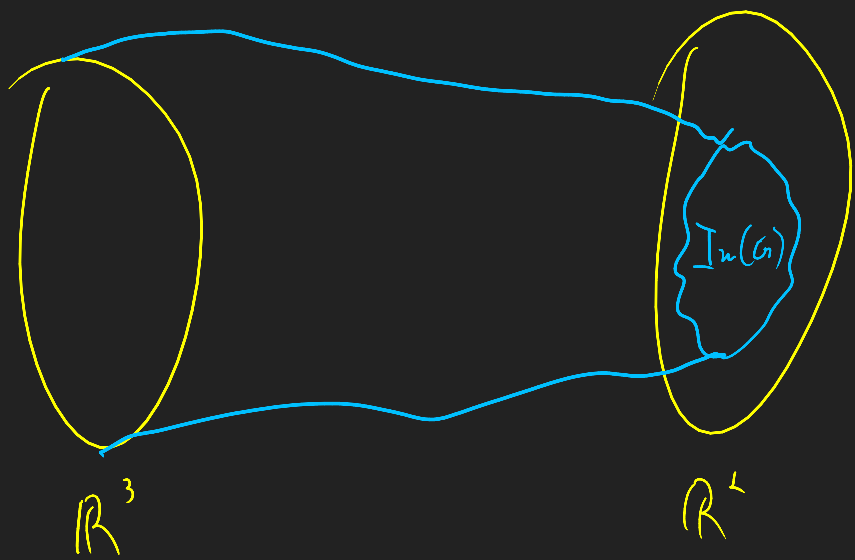
$$[G] = \begin{bmatrix} 1 & 2 & -4 \\ 2 & 3 & 1 \end{bmatrix}$$

$$[G(2,-5,4)] = [G][(2,-5,4)]$$

$$= \begin{bmatrix} 1 & 2 & -4 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -5 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 & -10 & -16 \\ 4 & -15 & 4 \end{bmatrix} = \begin{bmatrix} -24 \\ -7 \end{bmatrix}$$

$G(B) = \{ (1,2), (2,3), (-4,1) \}$ é gerador de $Im(G)$

$G(v) = a(1,2) + b(2,3) + c(-4,1)$



Im(G) é subespaço de $\mathbb{R}^2$

Dim($\mathbb{R}^2$) = 2

Dim(Im(G)) = 2

Im(G) = $\mathbb{R}^2$

G é sobrejetora

A IMAGEM DE UM ESPAÇO VETORIAL POR UMA TRANSF. LINEAR
É UM SUBESPAÇO VETORIAL DO CONTRADOMÍNIO

$C = \{ (1,-1,0), (2,2,0), (0,1,1) \}$

Base de $\mathbb{R}^3$

$$\begin{cases} G(1,-1,0) = (\ , \ ) \\ G(2,2,0) = (\ , \ ) \\ G(0,1,1) = (\ , \ ) \end{cases}$$

$G(x,y,z) = aG(1,-1,0) + bG(2,2,0) + cG(0,1,1)$

$D : P_2 \to P_2$ $\qquad D(p) = p'$ derivad.

$B = \{1, t, t^2\}$ base de P_2

$$\begin{cases} D(1) = 0 \\ D(t) = 1 \\ D(t^2) = 2t \end{cases}$$

$$D(2 - 3t + 4t^2) = 2D(1) - 3D(t) + 4D(t^2)$$
$$= 2(0) - 3(1) + 4(2t)$$
$$= -3 + 8t.$$

$T : \mathbb{R}^3 \to P_2$ $\qquad T(x, y, z) = (x+y) + (2z)t - zt^2$

$$\begin{cases} T(1,0,0) = 1 \\ T(0,1,0) = 1 \\ T(0,0,1) = 2t - t^2 \end{cases}$$

$$T(3,-2,5) = 3(1) - 2(1) + 5(2t - t^2) = 1 + 10t - 5t^2$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 3-2 \\ 10 \\ -5 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -5 \end{bmatrix}$$

$1 + 10t - 5t^2$

$H: \mathbb{R} \to \mathbb{R}^3 \qquad H(t) = (2t, t^2, 3t+5)$

a) H é linear?

$H(0) = (2(0), 0^2, 3(0)+5) = (0,0,5) \neq (0,0,0) = 0_{\mathbb{R}^3}$

Não é Linear

$H(u+v) \overset{?}{=} H(u) + H(v)$?

$H(\lambda u) \overset{?}{=} \lambda H(u)$

$H(u+v) = (2(u+v), (u+v)^2, 3(u+v)+5)$

$= (2u+2v, u^2+2uv+v^2, 3u+3v+5)$ //

$H(u)+H(v) = (2u, u^2, 3u+5) + (2v, v^2, 3v+5)$

$= (2u+2v, u^2+v^2+0, 3u+3v+5+5)$ $\neq$

Não é aditiva logo não é linear.

$H(\lambda u) = \lambda H(u)$? Exercício.