Coordenadas. EXEMPLOS

Espaço de matrizes $M_{2\times 2}(\mathbb{R})$

Sejam $B = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right\}$

e $C = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$

bases de $M_{2\times 2}$.

Escreva as matrizes de coordenadas dos seguintes vetores:

$u = \begin{pmatrix} 1 & -3 \\ 5 & 7 \end{pmatrix}$ $\quad v = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

$[u]_B$; $[u]_C$ $\quad [v]_B$, $[v]_C$.

$[u]_C = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$; $\text{coord.}(u) = (1, -3, 5, 7) \in \mathbb{R}^4$

$u = \begin{pmatrix} 1 & -3 \\ 5 & 7 \end{pmatrix} = a\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + c\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + d\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & -3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{cases} a = 1 \\ b = -3 \\ c = 5 \\ d = 7 \end{cases} \Rightarrow [u]_C = \begin{bmatrix} 1 \\ -3 \\ 5 \\ 7 \end{bmatrix}_C$

$$[u]_B = \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{bmatrix}_B \qquad u = \begin{pmatrix} 1 & -3 \\ 5 & 7 \end{pmatrix}$$

$$u = \alpha v_1 + \beta v_2 + \gamma v_3 + \delta v_4$$

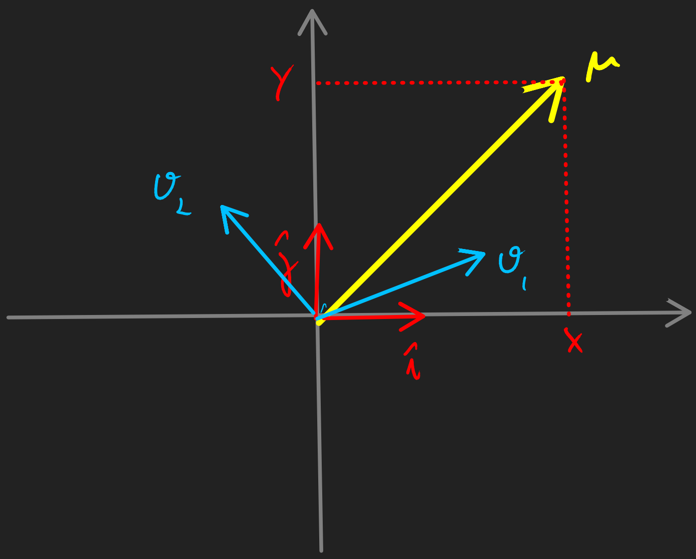
$$B = \left\{ \underset{v_1}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}, \underset{v_2}{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}, \underset{v_3}{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}, \underset{v_4}{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}} \right\}$$

$$\begin{pmatrix} 1 & -3 \\ 5 & 7 \end{pmatrix} = \alpha \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \beta \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \gamma \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \delta \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} \alpha + \gamma & \beta - \delta \\ \beta + \delta & \alpha - \gamma \end{pmatrix}$$

$$\begin{cases} \alpha + \gamma = 1 \\ \beta - \delta = -3 \\ \beta + \delta = 5 \\ \alpha - \gamma = 7 \end{cases}$$

$$\begin{cases} 2\alpha = 8 \\ 2\beta = 2 \\ 2\gamma = -6 \\ 2\delta = 8 \end{cases} \Rightarrow \begin{aligned} \alpha &= 4 \\ \beta &= 1 \\ \gamma &= -3 \\ \delta &= 4 \end{aligned} \qquad [u]_B = \begin{bmatrix} 4 \\ 1 \\ -3 \\ 4 \end{bmatrix}$$



$[u]_c = \begin{bmatrix} x \\ y \end{bmatrix}$

$u = x\hat{i} + y\hat{j}$

$u = a v_1 + b v_2$

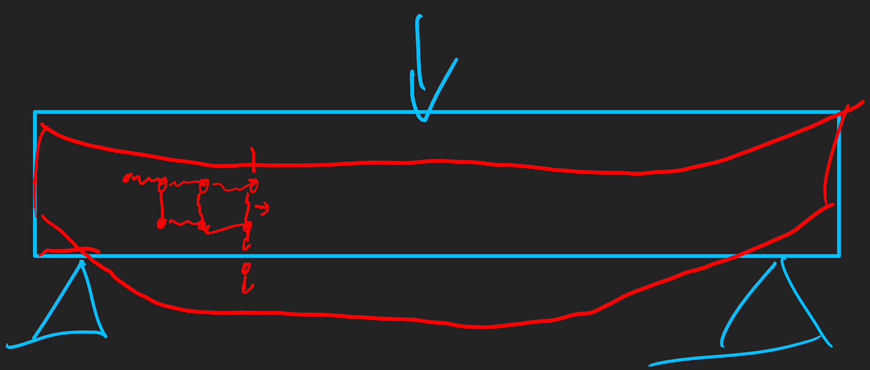
$B = \{v_1, v_2\}$ é base

$[u]_B = \begin{bmatrix} a \\ b \end{bmatrix}$

$\mathbb{R}^2$: $C = \{(1,0), (0,1)\}$

$[(x,y)]_C = \begin{bmatrix} x \\ y \end{bmatrix}$

$(x,y) = x(1,0) + y(0,1)$



Análise de elementos finitos.

Matriz de mudança de base

B, C bases do espaço V. $\dim(V) = n$

$u \in V$

$[u]_B$, $[u]_C$

$$[u]_B = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \quad [u]_C = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}_C = \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1n} \\ \vdots & \ddots & \vdots \\ \alpha_{n1} & \cdots & \alpha_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}_B \leftarrow$$

$$[u]_C = [I]^B_C \, [u]_B$$

A descobrir

destino

origem

conhecido

$$[I]^B_C = \begin{bmatrix} [v_1]_C & \cdots & [v_n]_C \end{bmatrix}$$

onde $B = \{v_1, \ldots, v_n\}$

Operador:

$$[u]_C = [I]_C^B \, [u]_B$$

$$[u]_C = [\underset{\substack{\uparrow \\ \text{identidade}}}{I}]_C^B \, [u]_B$$

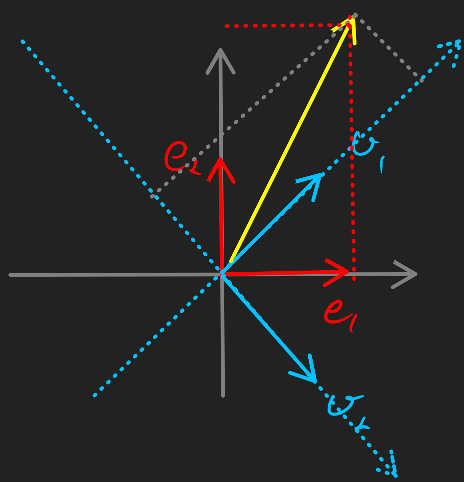
$$\boxed{u = I(u)}$$

Exemplo:

$\mathbb{R}^2$ bases:

$$B = \{\underset{v_1}{(1,1)}, \underset{v_2}{(1,-1)}\}$$

$$C = \{\underset{e_1}{(1,0)}, \underset{e_2}{(0,1)}\}$$



Obter $[I]_C^B$ $[I]_B^C$

$$[I]_C^B = \begin{bmatrix} [v_1]_C & [v_2]_C \end{bmatrix}$$

$$[v_1]_C = [(1,1)]_C = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow [I]_C^B = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$[v_2]_C = [(1,-1)]_C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

v t.q. que $[v]_B = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$

$$[v]_C = [I]_C^B [v]_B$$
$$= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$
$$[v]_C = \begin{bmatrix} 3 \\ -7 \end{bmatrix} \Rightarrow v = 3(1,0) - 7(0,1)$$
$$= (3,-7).$$

Se $[v]_B = \begin{bmatrix} -2 \\ 5 \end{bmatrix} \Rightarrow v = -2(1,1) + 5(1,-1)$
$$= (-2+5, -2-5)$$
$$v = (3, -7)$$

$$[I]_B^C = \big[[e_1]_B , [e_2]_B \big] = \big[[(1,0)]_B \quad [(0,1)]_B \big]$$

$[e_1]_B$: $(1,0) = \alpha v_1 + \beta v_2$
$$= \alpha(1,1) + \beta(1,-1)$$
$$(1,0) = (\alpha+\beta, \alpha-\beta)$$

$$\begin{cases} \alpha + \beta = 1 \\ \alpha - \beta = 0 \end{cases} \Rightarrow \begin{cases} 2\alpha = 1 \\ 2\beta = 1 \end{cases} \Rightarrow \begin{matrix} \alpha = 1/2 \\ \beta = 1/2 \end{matrix} \Rightarrow [e_1]_B = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

$[e_2]_B$:

$$(0,1) = \alpha(1,1) + \beta(1,-1)$$

$$\begin{cases} \alpha + \beta = 0 \\ \alpha - \beta = 1 \end{cases} \Rightarrow \begin{matrix} 2\alpha = 1 \\ 2\beta = -1 \end{matrix} \Rightarrow [e_2]_B = \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}$$

$$[I]_C^B = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix}.$$

Escreva o vetor u = (4, -3) na base B.

$$[u]_C = \begin{bmatrix} 4 \\ -3 \end{bmatrix} \quad \text{quero } [u]_B = [I]_B^C [u]_C$$

$$[u]_B = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 4/2 - 3/2 \\ 4/2 - \left(\frac{-3}{2}\right) \end{bmatrix} = \begin{bmatrix} 1/2 \\ 7/2 \end{bmatrix} \checkmark$$

$$u = \frac{1}{2}(1,1) + \frac{7}{2}(1,-1) = \left(\frac{1}{2} + \frac{7}{2}, \frac{1}{2} - \frac{7}{2}\right)$$
$$= \left(\frac{8}{2}, \frac{-6}{2}\right) = (4,-3) \checkmark$$