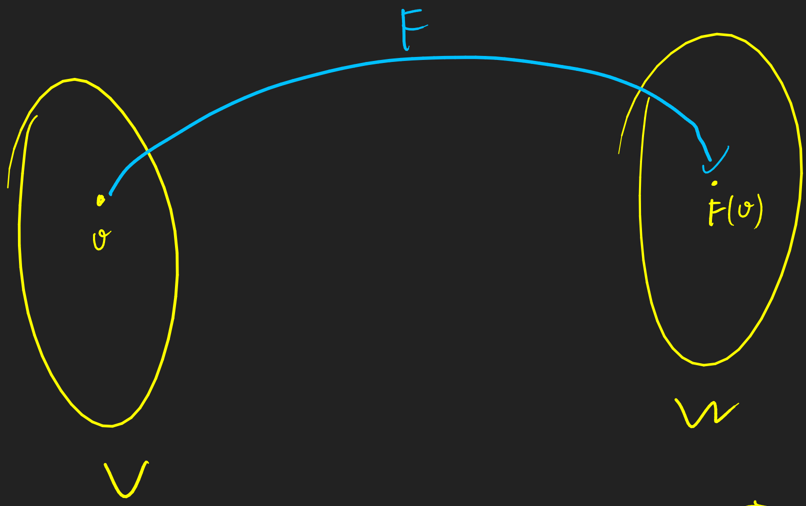
$F : V \to W$ transf. linear

V, W esp. vetoriais sobre $\mathbb{R}$



$F(v) \in W$

↳ vetor de W

(imagem de v por F)

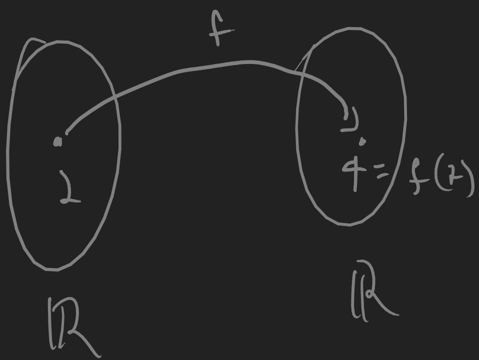
$B = \{v_1, \dots, v_n\}$ base de $V \Rightarrow \dim(V) = n$

$C = \{w_1, \dots, w_m\}$ base de $W \Rightarrow \dim(W) = m$

$f : \mathbb{R} \to \mathbb{R}$ $\quad f(x) = x^2$

$f(2) = 4$

↳ 4



$v = \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n \Rightarrow$

$$[v]_B = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}$$

→ Escalares (números reais)

MATRIZ de coordenadas

$F(v) = \beta_1 w_1 + \beta_2 w_2 + \dots + \beta_m w_m \in W$

$$\underbrace{[\underbrace{F(v)}_{\text{vetor}}]_C}_{\text{matriz}} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{bmatrix}$$

$[v]_D = [I]_D^B [v]_B$

Matriz da transf.

$$[F(v)]_C = [F]_{\underset{\text{CD}}{C}}^{\overset{\text{dom}}{B}} [v]_B$$

bases

$$\begin{bmatrix} \vdots \end{bmatrix}_{m\times 1} = \begin{bmatrix} \cdots \\ \vdots & \ddots \end{bmatrix}_{m\times n} \begin{bmatrix} \vdots \end{bmatrix}_{n\times 1}$$

n colunas — n linhas

m linhas

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dados F, B, C, obter $[F]_C^B$

$B = \{v_1, \dots, v_n\}$

Calcular $F(B)$:

vetor de W $= F(v_1) = \alpha_{11} w_1 + \alpha_{21} w_2 + \dots + \alpha_{m1} w_m$ na base C

$\vdots$ calcular

$F(v_n) = \alpha_{1n} w_1 + \dots + \alpha_{mn} w_m$

$$[F(v_1)]_C = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \\ \vdots \\ \alpha_{m1} \end{bmatrix} \rightarrow \text{matriz de coord.}$$

As colunas de $[F]_C^\beta$ são $[F(v_i)]_C$

$$[F]_C^B = \left[[F(v_1)]_C \cdots\cdots [F(v_n)]_C \right]_{m \times n}$$

$$[F]_C^B = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn} \end{bmatrix}$$

dado $[v]_B = \alpha_1 v_1 + \cdots + \alpha_n v_n$

$$\begin{bmatrix} \beta_1 \\ \vdots \\ \beta_m \end{bmatrix} = [F(v)]_C = \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1n} \\ \vdots & \ddots & \vdots \\ \alpha_{m1} & \cdots & \alpha_{mn} \end{bmatrix}_{m \times n} \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix}_{n \times 1}$$

– coordenadas do vetor $F(v) \in W$ na base C. //

Exemplo:

$$B = \{ u_1 = (1,1),\ u_2 = (1,2) \}$$

base de $\mathbb{R}^2$

Transf. lineares: $S : \mathbb{R}^2 \to \mathbb{R}^2$
$T : \mathbb{R}^2 \to \mathbb{R}^2$

$$S(x,y) = (x+2y,\ 4x)$$
$$T(x,y) = (y,\ x+3y)$$

1) dado um vetor (x, y) qualquer,

escreva as coordenadas de (x,y) na base B: [(x,y)]

$$[(x,y)]_B = \begin{bmatrix} a \\ b \end{bmatrix}_B$$ tal que $(x,y) = a u_1 + b u_2$

$$(x,y) = a(1,1) + b(1,2)$$ equação vetorial

$$= (a+b,\ a+2b)$$

$$\begin{cases} a + b = x \\ a + 2b = y \end{cases} \Rightarrow \begin{cases} a = x - b \\ x - b + 2b = y \Rightarrow b = y - x \end{cases}$$

$$a = x - b = x - (y - x) = 2x - y.$$

$$\boxed{[(x,y)]_B = \begin{bmatrix} 2x - y \\ -x + y \end{bmatrix}}$$ coordenadas de (x,y)

Note:

$$M_B : \mathbb{R}^2 \to M_{2\times1} :$$
$$M_B(x,y) = \begin{bmatrix} 2x - y \\ -x + y \end{bmatrix}$$

Obtenção da matriz de coordenadas de um vetor em relação à base B é uma transformação linear BIJETORA

b) Obtenha [S], a matriz da transformação S, em relação à base B

$$[S]_B^B = [S]_B = [S]$$

operador/transf.

base B está subentendida

$B = \{ u_1 = (1,1),\ u_2 = (1,2) \}.$ $\quad S(x,y) = (x+2y,\ 4x)$

objetivo: $[S] = \begin{bmatrix} [S(u_1)] & [S(u_2)] \end{bmatrix}$

$S(u_1) = S(1,1) = (1+2(1),\ 4(1)) = (3,4)$

$S(u_2) = S(1,2) = (1+2(2),\ 4(1)) = (5,4)$

$[S(u_1)]_B = [\underbrace{(3,4)}_{\text{vetor}}] = \underbrace{\begin{bmatrix} 2(3)-4 \\ -3+4 \end{bmatrix}}_{\text{coord. } B} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$[S(u_2)] = [(5,4)] = \begin{bmatrix} 2(5)-4 \\ -5+4 \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \end{bmatrix}$

$[S] = \begin{bmatrix} 2 & 6 \\ 1 & -1 \end{bmatrix}$ matriz de S na base B.

c) Obtenha a matriz [T] na base B $\quad T(x,y) = (y,\ x+3y)$

$T(u_1) = T(1,1) = (1,\ 1+3) = (1,4)$

$T(u_2) = T(1,2) = (2,\ 1+3(2)) = (2,7)$

$[T(u_1)] = [(1,4)] = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$[T(u_2)] = [(2,7)] = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$

verificando.

$\Rightarrow (2,7) = -3(1,1) + 5(1,2)$

$(2,7) = (-3+5,\ -3+10)$ ✓

$$[T] = \begin{bmatrix} -2 & -3 \\ 3 & 5 \end{bmatrix}.$$

$$[S] = \begin{bmatrix} 2 & 6 \\ 1 & -1 \end{bmatrix}, \quad [T] = \begin{bmatrix} -2 & -3 \\ 3 & 5 \end{bmatrix} //$$

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d) Seja $v = (5, 3)$ $\quad S(x,y) = (x+2y, 4x)$

Calcule $[S(v)]_B$

2 formas:

$\rightarrow$ $S(v) = S(5,3) = (5+2(3), 4(5)) = (11, 20)$

$$[S(v)] = [(11,20)] = \begin{bmatrix} 2(11) - 20 \\ -11 + 20 \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \end{bmatrix} \qquad [(x,y)]_B = \begin{bmatrix} 2x - y \\ -x + y \end{bmatrix}$$

Altern:

$$[(5,3)] = \begin{bmatrix} 2(5) - 3 \\ -5 + 3 \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$$

$$[S(5,3)] = [S]\,[(5,3)]$$

$$= \begin{bmatrix} 2 & 6 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 7 \\ -2 \end{bmatrix} = \begin{bmatrix} 2(7) - 2(6) \\ 7 - (-2) \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \end{bmatrix} \checkmark$$

e) Obtenha [(S+T)], a matriz da transformação (S+T)

$$(S+T)(v) = S(v) + T(v)$$

adição no espaço vetorial das transformações

adição no espaço vetorial W: $\mathbb{R}^2$

$$[(S+T)] = \Big[\, \underbrace{[(S+T)(u_1)]}_{S(u_1)+T(u_1)} \quad [(S+T)(u_2)] \,\Big]$$

$$(S+T)(u_1) = S(u_1) + T(u_1) = (3,4) + (1,4) = (4,8)$$

$$(S+T)(u_2) = S(u_2) + T(u_2) = (5,4) + (2,7) = (7,11)$$

Coordenadas

$$[(x,y)]_B = \begin{bmatrix} 2x-y \\ -x+y \end{bmatrix}$$

$$[(S+T)(u_1)] = [(4,8)] = \begin{bmatrix} 2(4)-8 \\ -4+8 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Analogamente:

$$[(S+T)(u_1)] = [S(u_1) + T(u_1)] = [S(u_1)] + [T(u_1)]$$

$$= \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$[(S+T)(u_2)] = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$ (exercício)

$$[S+T] = \begin{bmatrix} 0 & 3 \\ 4 & 4 \end{bmatrix}$$

Note: $[S] = \begin{bmatrix} 2 & 6 \\ 1 & -1 \end{bmatrix}$, $[T] = \begin{bmatrix} -2 & -3 \\ 3 & 5 \end{bmatrix}$ //

$S \in L(\mathbb{R}^2)$

$[S] \in M_{2\times 2}(\mathbb{R})$

$M_B : L(\mathbb{R}^2) \to M_{2\times 2}$

↳ Transf. linear

$$\underbrace{M_B(S+T)}_{[S+T]} = \underbrace{M_B(S)}_{[S]} + \underbrace{M_B(T)}_{[T]}$$

$$[S+T] = [S] + [T]$$

$$[S+T] = \begin{bmatrix} 2 & 6 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} -2 & -3 \\ 3 & 5 \end{bmatrix}$$

adição em $L(\mathbb{R}^2)$
adição de transformações

adição de matrizes.

$$= \begin{bmatrix} 2-2 & 6-3 \\ 1+3 & -1+5 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 4 & 4 \end{bmatrix} \checkmark$$

f) Obtenha [(S+T) (5,3)]

$$[5,3] = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$$

$$[(S+T)(5,3)] = [(S+T)][(5,3)] = \begin{bmatrix} 0 & 3 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} 7 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0(7)+3(-2) \\ 4(7)+4(-2) \end{bmatrix} = \begin{bmatrix} -6 \\ 20 \end{bmatrix}$$

$$\left[(S+T)(5,3)\right] = \begin{bmatrix} -6 \\ 20 \end{bmatrix}$$

Logo $(S+T)(5,3) = -6\,u_1 + 20\,u_2$

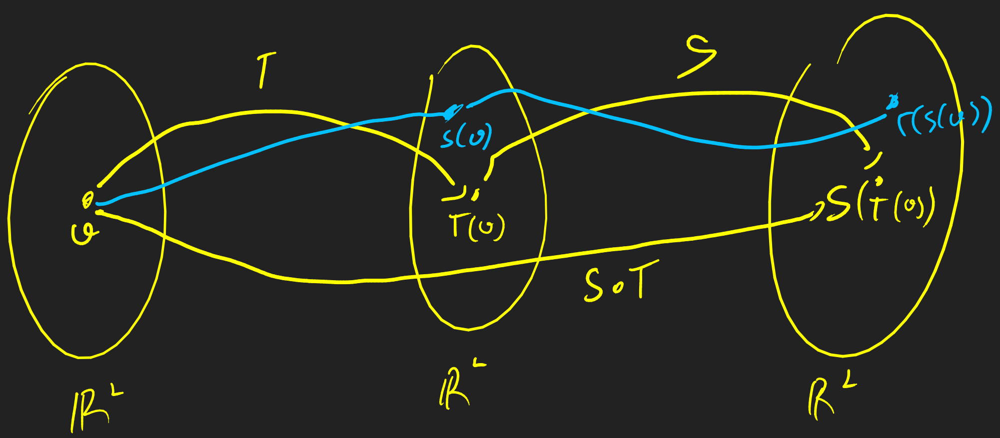
$= -6(1,1) + 20(1,2)$

$= (14, 34)$

verifique aplicando a transf. diretamente.

g) Obtenha [S o T]

$$(S \circ T)(v) = S(T(v))$$



$$[S \circ T] = \left[\ \underbrace{[(S \circ T)(u_1)]}_{S(T(u_1))} \quad \underbrace{[(S \circ T)(u_2)]}_{S(T(u_2))}\ \right] \qquad B = \{u_1, u_2\}$$

$$[T(u_1)] = [T][u_1] = [T]\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$u_1 = 1\,u_1 + 0\,u_2 \qquad [u_1] = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$u_2 = 0\,u_1 + 1\,u_2 \qquad [u_2] = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$[S(v_1)] = [S][v_1]$$

se $v_1 = T(u_1)$

$$[v_1] = [T(u_1)] = [T][u_1]$$

$$[S(v_1)] = [S][v_1] = ([S]\ [T])[u_1]$$

produto de matrizes.

$$[(S\circ T)(u_1)] = [S(T(u_1))] = [S]\,[T(u_1)] = [S][T][u_1]$$

$$[S] = \begin{bmatrix} 2 & 6 \\ 1 & -1 \end{bmatrix}, \quad [T] = \begin{bmatrix} -2 & -3 \\ 3 & 5 \end{bmatrix}$$

$$[(S\circ T)(u_1)] = [S][T][u_1] = \left(\underbrace{\begin{bmatrix} 2 & 6 \\ 1 & -1 \end{bmatrix}}\underbrace{\begin{bmatrix} -2 & -3 \\ 3 & 5 \end{bmatrix}}\right)\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \left(\begin{bmatrix} -4+18 & -6+30 \\ -2-3 & -3-5 \end{bmatrix}\right)\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 24 \\ -5 & -8 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 14 \\ -5 \end{bmatrix}$$

$$[(S\circ T)(u_2)] = [S][T][u_2] = \begin{bmatrix} 2 & 6 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -2 & -3 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 24 \\ -5 & -8 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 24 \\ -8 \end{bmatrix}$$

$$[(S\circ T)(u_1)] = [S][T][u_1] = \begin{bmatrix}2 & 6\\ 1 & -1\end{bmatrix}\begin{bmatrix}-2 & -3\\ 3 & 5\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$[(S\circ T)(u_1)] = [S\circ T]\cdot[u_1]$$

$$[S\circ T] = [S][T]$$

$$(S\circ T)(x,y) = S(T(x,y))$$
$$= S(y,\ x+3y)$$
$$= S(\sim x + \sim y,\ \sim x + \sim y)$$

$$[(S\circ T)(1,1)] = [(\sim,\ \sim)] = \begin{bmatrix}14\\ -5\end{bmatrix}$$

matriz da adição de transformações = adição das matrizes de transformações
matriz da multiplicação da transf. por escalar = multiplicação da matriz da trannsf. por escalar
matriz da composição = produto das matrizes

$$[S \circ T] = \begin{bmatrix} 14 & 24 \\ -5 & -8 \end{bmatrix}$$