1) Obtenha uma base em que a forma quadrática abaixo seja escrita como soma de quadrados

$$q(x, y) = x^2 - 10xy + y^2$$

Termo cruzado

$$q(a,b) = \lambda_1 a^2 + \lambda_2 b^2$$

$$q: \mathbb{R}^2 \to \mathbb{R}$$

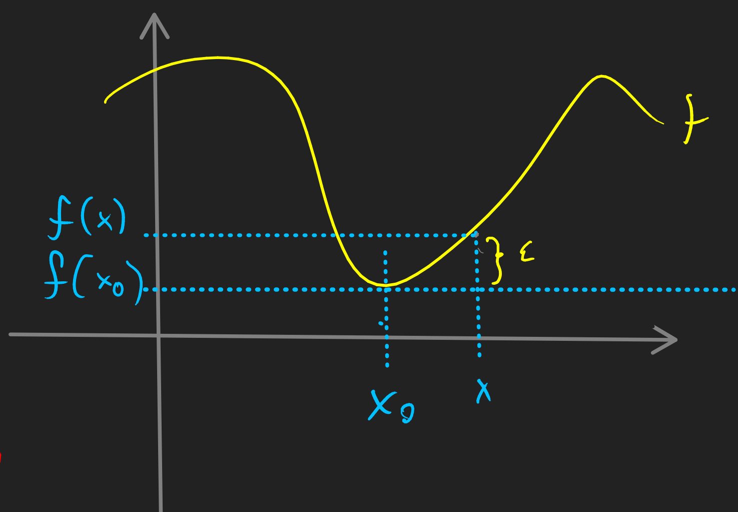
esp. vetorial

escalar

Parênteses:

$f: \mathbb{R} \to \mathbb{R}$ (cálculo)

Série de Taylor



$$f(x) \approx f(x_0) + \cancel{f'(x_0)}(x-x_0) + \frac{f''(x_0)}{2}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + \dots$$

0

Para x_0 ponto de máx/mínimo de f:

$$f(x) \approx f(x_0) + \frac{f''(x_0)}{2}(x-x_0)^2$$

$$f(\tilde{x}) \approx A + B\tilde{x}^2 \qquad \tilde{x} = (x - x_0)$$

$$g(\tilde{x}) = f(\tilde{x}) - A$$

$$g(\tilde{x}) = B\tilde{x}^2$$

em 2 variáveis:

$$g(x,y) \approx Ax^2 + Bxy + Cy^2$$

$$q(x, y) = x^2 - 10xy + y^2$$

$$g(x,y) = \begin{bmatrix} x & y \end{bmatrix} \underbrace{\begin{bmatrix} A & B/2 \\ B/2 & C \end{bmatrix}}_{\text{Simétrica}} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$q(x,y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} A & B \\ 0 & C \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

escolho =>

Diagonalizável numa base ortonormal

$$\begin{cases} q(a,b) = \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \lambda_1 a^2 + \lambda_2 b^2 \end{cases}$$

onde $[v]_c = \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow [v]_B = \begin{bmatrix} a \\ b \end{bmatrix}$

$$q(x,y) = \underset{A}{x^2} - \underset{B}{10xy} + \underset{C}{y^2}$$

$$[q]_c = \begin{bmatrix} 1 & -5 \\ -5 & 1 \end{bmatrix}$$

Diagonalizar [q]

$$p_q(\lambda) = \det\begin{bmatrix} 1-\lambda & -5 \\ -5 & 1-\lambda \end{bmatrix} = (1-\lambda)^2 - 25$$

$$= \lambda^2 - 2\lambda + 1 - 25$$

$$= \lambda^2 - 2\lambda - 24 \Rightarrow \begin{cases} \lambda_1 \lambda_2 = -24 \\ \lambda_1 + \lambda_2 = 2 \end{cases}$$

$$\lambda_1 = 6$$

$$\lambda_2 = -4$$

$$[q] = \begin{bmatrix} 6 & 0 \\ 0 & -4 \end{bmatrix} \Rightarrow q(a,b) = \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 6 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$q(a,b) = 6a^2 - 4b^2$$

Autovetores:

$$\lambda_1 = 6 \qquad \begin{bmatrix} 1 & -5 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 6 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{cases} x - 5y = 6x \\ -5x + y = 6y \end{cases} \Rightarrow \begin{matrix} -5x - 5y = 0 \\ x = -y \end{matrix}$$

$$v_1 = (x, -x) = x(1, -1)$$

Escolho o autovetor de norma 1 : $v_1 = \frac{1}{\sqrt{2}}(1,-1)$ //

$\lambda_2 = -4$

$$\begin{bmatrix} 1 & -5 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = -4 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{cases} x - 5y = -4y \\ -5x + y = -4x \end{cases} \Rightarrow 5x - 5y = 0$$

$$x = y$$

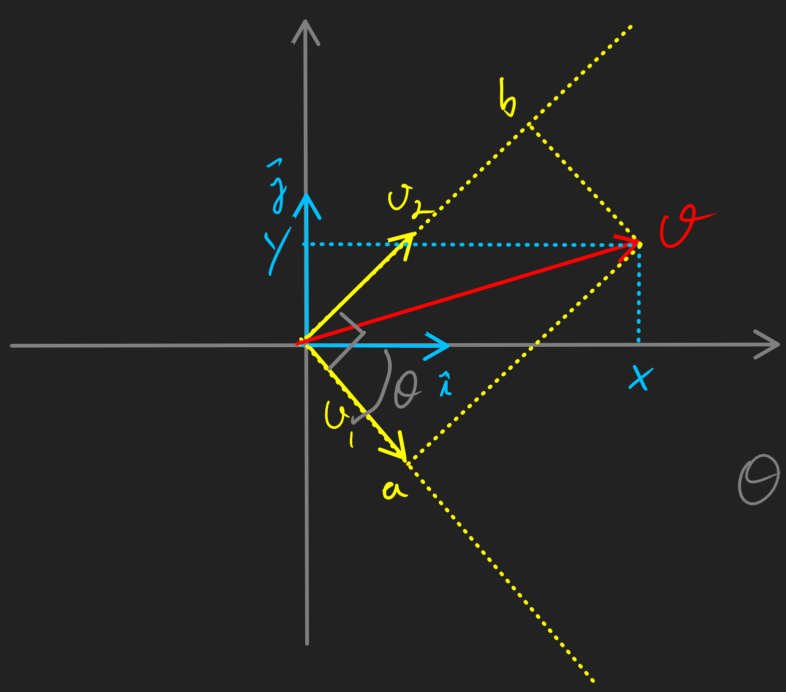
$$v_2 = (x, x) = x(1,1)$$

Escolho autovetor normalizado: $v_2 = \frac{1}{\sqrt{2}}(1,1)$

$$B = \left\{ v_1 = \frac{1}{\sqrt{2}}(1,-1),\ v_2 = \frac{1}{\sqrt{2}}(1,1) \right\}$$

é a bse em que [q] é diagonal

$q(x,y) = x^2 - 10xy + y^2$ $\qquad$ $q(a,b) = 6a^2 - 4b^2$



$\theta = \pi/4$

matriz mudança de base:

$$[I]_C^B = \begin{bmatrix} [v_1]_C & [v_2]_C \end{bmatrix}$$

$$B = \left\{ v_1 = \frac{1}{\sqrt{2}}(1,-1),\ v_2 = \frac{1}{\sqrt{2}}(1,1) \right\}$$

$$[I]_C^B = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

Ortogonal

$$\begin{bmatrix} x \\ y \end{bmatrix}_C = [I]_C^B \begin{bmatrix} a \\ b \end{bmatrix}_B = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{cases} x = \frac{1}{\sqrt{2}}(a+b) \\ y = \frac{1}{\sqrt{2}}(a-b) \end{cases}$$

ortonormais

$$[I]_B^C = \left([I]_C^B\right)^{-1} \stackrel{\vee}{=} \left([I]_C^B\right)^T = \left(\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\right)^T$$

$$[I]_B^C = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{cases} a = \frac{1}{\sqrt{2}}(x-y) \\ b = \frac{1}{\sqrt{2}}(x+y) \end{cases}$$

$$q(x,y) = x^2 - 10xy + y^2 \iff q(a,b) = 6a^2 - 4b^2$$

$$q(a,b) = 6a^2 - 4b^2$$
$$= 6\left(\frac{1}{\sqrt{2}}(x-y)\right)^2 - 4\left(\frac{1}{\sqrt{2}}(x+y)\right)^2$$
$$= \frac{6}{2}(x-y)^2 - \frac{4}{2}(x+y)^2$$
$$= 3(x^2 - 2xy + y^2) - 2(x^2 + 2xy + y^2)$$

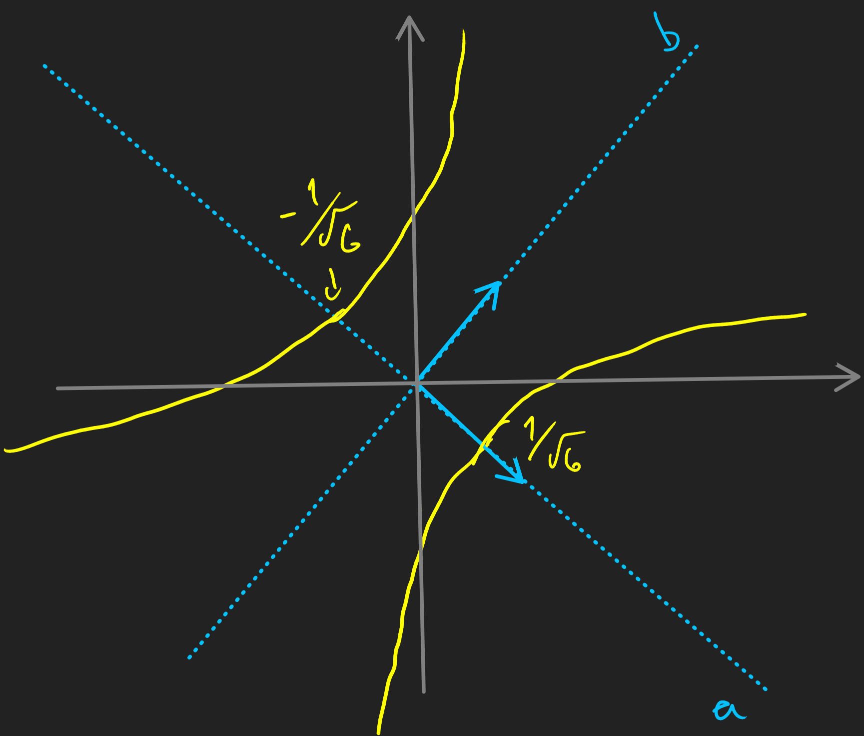
$$q(x,y) = x^2 - 10xy + y^2$$

Curvas de nível:

Obtenha a curva de nível q(x,y) = 1

$$x^2 - 10xy + y^2 = 1 \quad \text{ou} \quad \boxed{6a^2 - 4b^2 = 1}$$

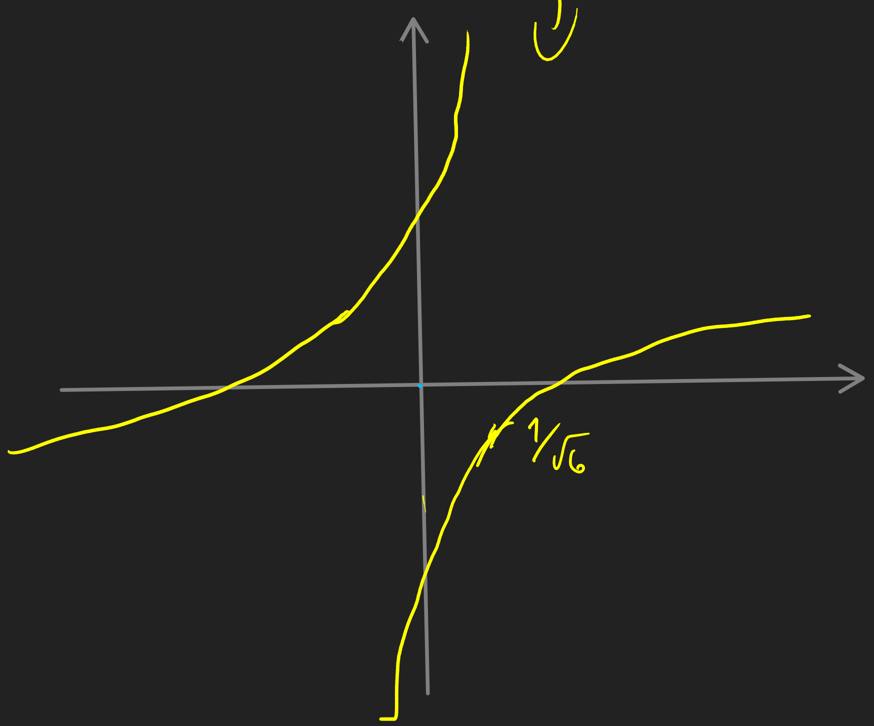
$$\frac{a^2}{(1/6)} - \frac{b^2}{(1/4)} = 1$$

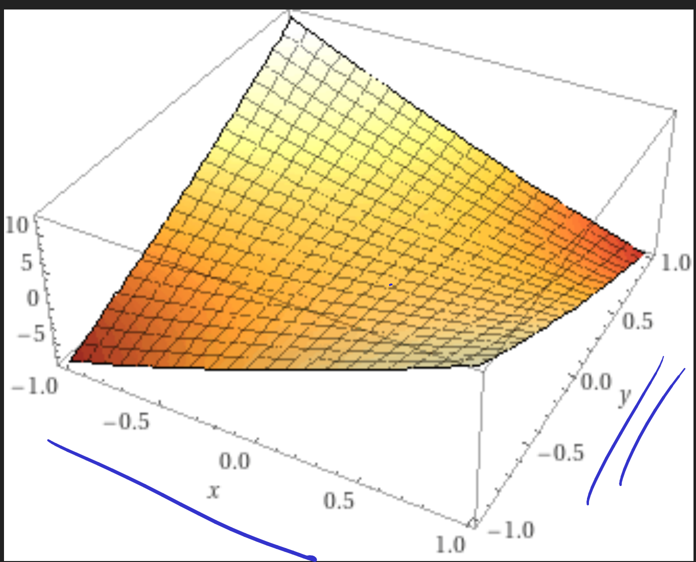


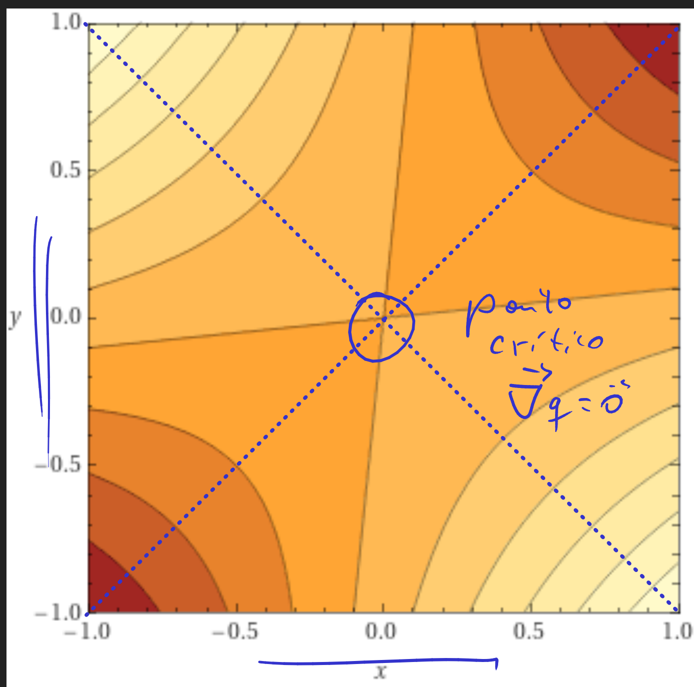
Hipérbole!!!

Origem é ponto de sela de $q(x,y) = x^2 - 10xy + y^2$

$$q(x,y) = x^2 - 10xy + y^2 = 1$$







Hessiana: $\Longleftrightarrow$ forma quadrática

$$H(q(x,y)) = \begin{bmatrix} \dfrac{\partial^2 q}{\partial x^2} & \dfrac{\partial^2 q}{\partial x \partial y} \\[2ex] \dfrac{\partial^2 q}{\partial y \partial x} & \dfrac{\partial^2 q}{\partial y^2} \end{bmatrix}$$