Coordenadas, exemplos.

Espaço de matrizes. $M_{2\times 2}(\mathbb{R})$

Sejam

$$B = \left\{ \underset{v_1}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}, \underset{v_2}{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}, \underset{v_3}{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}, \underset{v_4}{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}} \right\}$$

e

$$C = \left\{ \underset{e_1}{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}, \underset{e_2}{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}, \underset{e_3}{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}}, \underset{e_4}{\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}} \right\}$$

bases de $M_{2\times 2}(\mathbb{R})$.

Obtenha as matrizes de coordenadas, em relação a cada uma das bases B e C, do vetor

$$u = \begin{pmatrix} 1 & -3 \\ 5 & 7 \end{pmatrix} \qquad [u]_B \; ; [u]_C$$

$$[u]_C = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \qquad u = a e_1 + b e_2 + c e_3 + d e_4$$

$$\begin{pmatrix} 1 & -3 \\ 5 & 7 \end{pmatrix} = a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + d \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

equação vetorial

$$\begin{cases} a = 1 \\ b = -3 \\ c = 5 \\ d = 7 \end{cases} \Rightarrow [u]_C = \begin{bmatrix} 1 \\ -3 \\ 5 \\ 7 \end{bmatrix}$$

$$\text{Coord}_C(u) := (1, -3, 5, 7)$$

$$B = \left\{ \overset{v_1}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}, \overset{v_2}{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}, \overset{v_3}{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}, \overset{v_4}{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}} \right\}$$

$$[u]_B = \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{bmatrix} \qquad u = \alpha v_1 + \beta v_2 + \gamma v_3 + \delta v_4$$

$$\begin{pmatrix} 1 & -3 \\ 5 & 7 \end{pmatrix} = \alpha \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \beta \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \gamma \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \delta \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

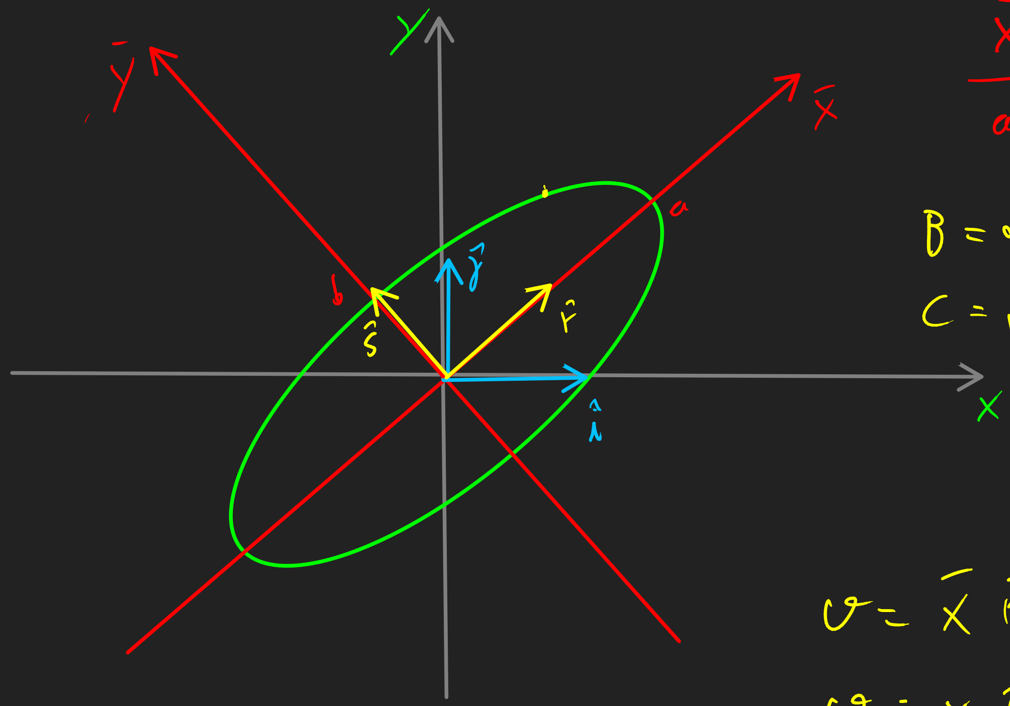
$$\begin{pmatrix} 1 & -3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} \alpha + \gamma & \beta - \delta \\ \beta + \delta & \alpha - \gamma \end{pmatrix}$$

$$\begin{cases} \alpha + \gamma = 1 \\ \beta - \delta = -3 \\ \beta + \delta = 5 \\ \alpha - \gamma = 7 \end{cases} \Rightarrow \begin{cases} 2\alpha = 8 \\ 2\beta = 2 \\ 2\gamma = -6 \\ 2\delta = 8 \end{cases} \Rightarrow \begin{cases} \alpha = 4 \\ \beta = 1 \\ \gamma = -3 \\ \delta = 4 \end{cases}$$

$$[u]_B = \begin{bmatrix} 4 \\ 1 \\ -3 \\ 4 \end{bmatrix} \Rightarrow u = 4 v_1 + 1 v_2 - 3 v_3 + 4 v_4.$$

$$4 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + 1 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - 3 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + 4 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 4-3 & 1-4 \\ 1+4 & 4-(-3) \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ 5 & 7 \end{pmatrix} \checkmark$$



$$\frac{\bar{x}^2}{a^2} + \frac{\bar{y}^2}{b^2} = 1$$

$B = \{\hat{r}, \hat{s}\} \leftarrow$

$C = \{\hat{i}, \hat{j}\}$

$$v = \bar{x}\,\hat{r} + \bar{y}\,\hat{s}$$

$$v = x\,\hat{i} + y\,\hat{j}$$

$$\begin{cases} \hat{r} = \alpha \hat{i} + \beta \hat{j} \\ \hat{s} = \gamma \hat{i} + \delta \hat{j} \end{cases}$$

Espaço dos polinômios

$$P_3(\mathbb{R})$$

$B = \{1,\ 1-t,\ (1-t)^2,\ (1-t)^3\} \leftarrow$ base

$\{1,\ 1-t,\ 1-2t+t^2,\ (1-2t+t^2)(1-t)\}$

$B = \{\underset{v_1}{1},\ \underset{v_2}{1-t},\ \underset{v_3}{1-2t+t^2},\ \underset{v_4}{1-3t+3t^2-t^3}\}$

$C = \{\overset{e_1}{1},\ \overset{e_2}{t},\ \overset{e_3}{t^2},\ \overset{e_4}{t^3}\} \leftarrow$ base

Obtenha $\boxed{[u]_B}$, $\boxed{[v]_B}$, $\boxed{[u]_C}$, $\boxed{[v]_C}$

? ? ?! ?

$u = 2 - 3t + t^2 + 2t^3$ $\quad$ vector $\longrightarrow$ $w = 3 - 2t - t^2$

$$[u]_c = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \leftarrow [u]_c$$

$$u = a \cdot 1 + bt + ct^2 + dt^3$$

$$2 - 3t + t^2 + 2t^3 = a \cdot 1 + bt + ct^2 + dt^3$$

$$\begin{cases} a = 2 \\ b = -3 \\ c = 1 \\ d = 2 \end{cases} \Rightarrow \boxed{[u]_c = \begin{bmatrix} 2 \\ -3 \\ 1 \\ 2 \end{bmatrix}}$$

$$[w]_c = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \qquad w = a e_1 + b e_2 + c e_3 + d e_4$$

$$3 - 2t - t^2 = a + bt + ct^2 + dt^3$$

$$\begin{cases} a = 3 \\ b = -2 \\ c = -1 \\ d = 0 \end{cases} \Rightarrow [w]_c = \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix}.$$

$$B = \{ \underset{v_1}{1}, \underset{v_2}{1-t}, \underset{v_3}{1-2t+t^2}, \underset{v_4}{1-3t+3t^2-t^3} \}$$

$$u = 2-3t+t^2+2t^3 \qquad w = 3-2t-t^2$$

$$[u]_B = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \qquad u = a v_1 + b v_2 + c v_3 + d v_4$$

$$2-3t+t^2+2t^3 = a \cdot 1 + b(1-t) + c(1-2t+t^2) + d(1-3t+3t^2-t^3)$$

$$2-3t+t^2+2t^3 = (a+b+c+d) + (-b-2c-3d)t + (c+3d)t^2 - dt^3$$

$$\begin{cases} a+b+c+d = 2 \\ -b-2c-3d = -3 \\ c+3d = 1 \\ -d = 2 \end{cases} \Rightarrow \begin{cases} a-5+7-2 = 2 \Rightarrow a = 2 \\ -b-14+6 = -3 \Rightarrow b = -5 \\ c-6 = 1 \Rightarrow c = 7 \\ d = -2 \end{cases}$$

$$[u]_B = \begin{bmatrix} 2 \\ -5 \\ 7 \\ -2 \end{bmatrix}$$

$$[w]_B = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \qquad w = a v_1 + b v_2 + c v_3 + d v_4$$

$$3-2t-t^2 = a\,1 + b(1-t) + c(1-2t+t^2) + d(1-3t+3t^2-t^3)$$

$$= (a+b+c+d) + (-b-2c-3d)t + (c+3d)t^2 - dt^3$$

$$\begin{cases} a + b + c + d = 3 \\ -b - 2c - 3d = -2 \\ c + 3d = -1 \\ -d = 0 \end{cases} \Rightarrow \begin{cases} a + 4 - 1 + 0 = 3 \Rightarrow a = 0 \\ -b + 2 = -2 \Rightarrow b = 4 \\ c = -1 \\ d = 0 \end{cases}$$

$$[v]_B = \begin{bmatrix} 0 \\ 4 \\ -1 \\ 0 \end{bmatrix}$$