Revisão, dúvidas produto interno

Lista de exercícios
Revisão 1.2 — Produto interno

1. Verifique quais dos 4 axiomas de espaço vetorial são satisfeitas pelas seguintes operações sobre os vetores $\boldsymbol{u} = (u_1, \ldots, u_n)$ e $\boldsymbol{v} = (v_1, \ldots, v_n)$ de $\mathbb{R}^n$:

(a) $\langle \boldsymbol{u}, \boldsymbol{v} \rangle = \sum_{i=1}^{n} u_i v_i$

↳ sigma: Σ σ

Somatório

$$\sum_{i=1}^{n} (u_i v_i) = u_1 v_1 + u_2 v_2 + u_3 v_3 + \cdots + u_n v_n$$

$$\langle u, v \rangle = \langle v, u \rangle$$

$$\langle u, v \rangle = \sum_{i=1}^{n} u_i v_i$$

$$\langle v, u \rangle = \sum_{i=1}^{n} \underbrace{v_i u_i}_{\text{comutativa}} = \sum_{i=1}^{n} u_i v_i = \langle u, v \rangle \checkmark$$

$$\langle u, v+w \rangle = \sum_{i=1}^{n} u_i (v_i + w_i) \quad \text{distributiva}$$

$$= \sum_{i=1}^{n} [u_i v_i + u_i w_i] = \sum_{i=1}^{n} u_i v_i + \sum_{i=1}^{n} u_i w_i$$

$$u_1 v_1 + u_1 w_1 + u_2 v_2 + u_2 w_2 + \cdots$$

$$\underbrace{u_1 v_1 + u_2 v_2 + \cdots + u_n v_n}_{} + \underbrace{u_1 w_1 + \cdots + u_n w_n}_{}$$

$$= \sum_{i=1}^{n} u_i v_i + \sum_{i=1}^{n} u_i w_i =$$

$$= \langle u, v \rangle + \langle v, w \rangle$$

(b) $\langle \boldsymbol{u}, \boldsymbol{v} \rangle = \prod_{i=1}^{n} u_i v_i = u_1 v_1 u_2 v_2 \cdots u_n v_n$

pi: $\prod$ π π

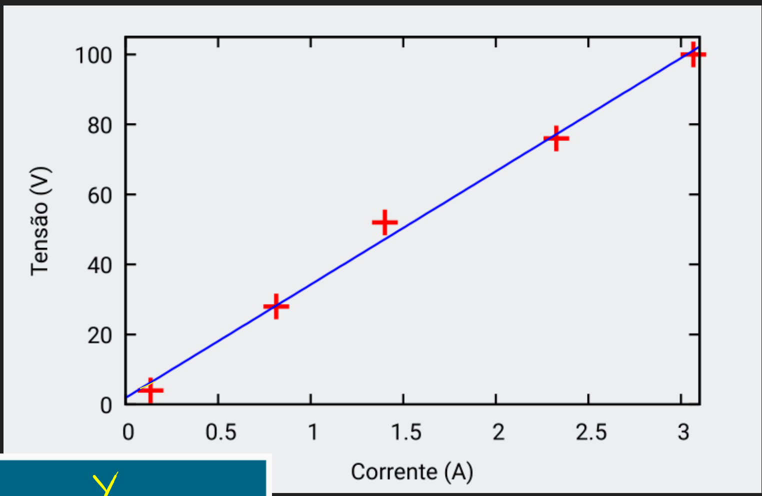
$$= \prod_{i=1}^{n} u_i v_i = (u_1 v_1)(u_2 v_2) \ldots (u_n v_n)$$

$$\langle u, v + w \rangle = \prod u_i (v_i + w_i) = \prod [u_i v_i + u_i w_i]$$

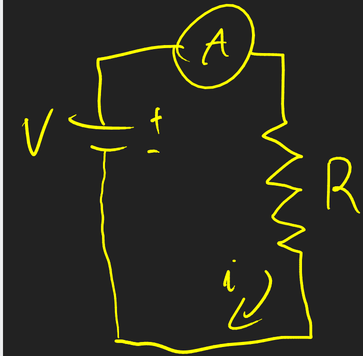
$$(u_1 v_1 + u_1 w_1)(u_2 v_2 + u_2 w_2) \cdots (u_n v_n + u_n w_n)$$

$$(u_1 v_1 u_2 w_2 + \ldots + \ldots + \ldots)$$

Método dos Mínimops Quadrados



$$V = Ri + V_0$$

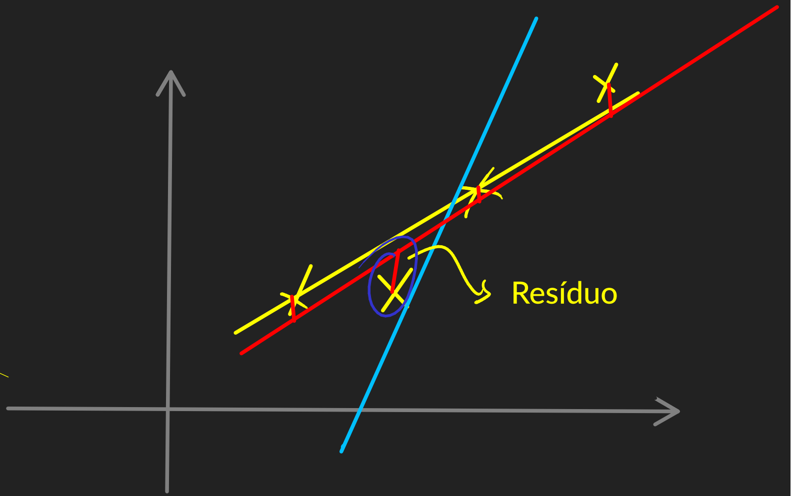


Dados $\quad x \quad y$

	Corrente i (A)	Tensão V (V)
x_1	0.135	4.00
x_2	0.814	28.0
x_3	1.400	52.0
x_4	2.327	76.0
x_5	3.066	100

$$V(i)$$

$$V(i) = V_0 + Ri + \cancel{\varepsilon(i)}$$



Espaço das funções:

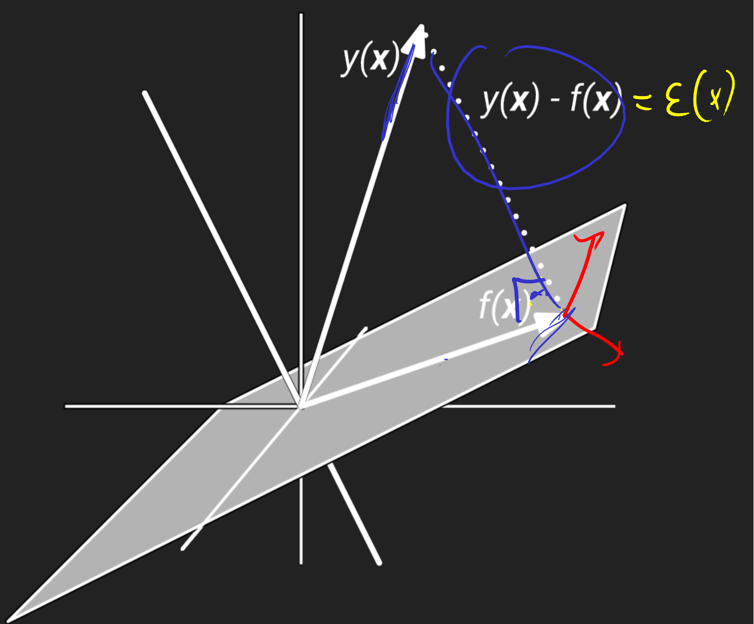
Função MEDIÇÃO/OBSERVAÇÃO: $y(x)$

Subespaço S dos polinômios $\quad y \notin S$

$$f(x) = \alpha_0 x^0 + \alpha_1 x^1 + \dots + \alpha_n x^n = \sum_{i=1}^{n} \alpha_i x^i$$

Base do subespaço:

$$B = \{1, x, x^2, \dots, x^n\}$$
$$\{v_1, v_2, v_3, \dots, v_n\}$$



$$y(x) = f(x) + \varepsilon(x)$$

$$\langle y(x) - f(x),\ f(x) \rangle = 0$$

$$\langle \underbrace{y(x) - \alpha_0 1 - \alpha_1 x - \dots \alpha_n x^n}, \ \alpha_0 1 + \alpha_1 x + \dots + \alpha_n x^n \rangle = 0$$

$$\begin{cases} \langle y - \alpha_1 \vartheta_1 - \alpha_2 \vartheta_2 \dots - \alpha_n \vartheta_n,\ \vartheta_1 \rangle = 0 \\ \langle y - \alpha_1 \vartheta_1 - \alpha_2 \vartheta_2 - \dots \alpha_n \vartheta_n,\ \vartheta_2 \rangle = 0 \\ \vdots \\ \langle y - \alpha_1 \vartheta_1 - \alpha_n \vartheta_2 \dots \alpha_n \vartheta_n,\ \vartheta_3 \rangle = 0 \end{cases}$$

$d_2 d_{a_3}$

$$\begin{cases} \alpha_1 \langle \vartheta_1, \vartheta_1 \rangle + \alpha_2 \langle \vartheta_2, \vartheta_1 \rangle + \dots \alpha_n \langle \vartheta_n, \vartheta_1 \rangle = \langle y, \vartheta_1 \rangle \\ \vdots \\ \alpha_1 \langle \vartheta_1, \vartheta_n \rangle + \alpha_2 \langle \vartheta_2, \vartheta_n \rangle + \dots \alpha_n \langle \vartheta_n, \vartheta_n \rangle = \langle y, \vartheta_n \rangle \end{cases}$$

$$\langle f, g \rangle = \sum_{i=1}^{n} f(x_i)\, g(x_i)$$

$$\langle f - \alpha_1 \vartheta_1 - \alpha_2 \vartheta_2 - \dots - \alpha_n \vartheta_n,\ \vartheta_1 \rangle = 0$$

$$\langle f, \vartheta_1 \rangle = \alpha_1 \langle \vartheta_1, \vartheta_1 \rangle + \alpha_2 \langle \vartheta_2, \vartheta_1 \rangle \dots + \alpha_n \langle \vartheta_n, \vartheta_1 \rangle$$

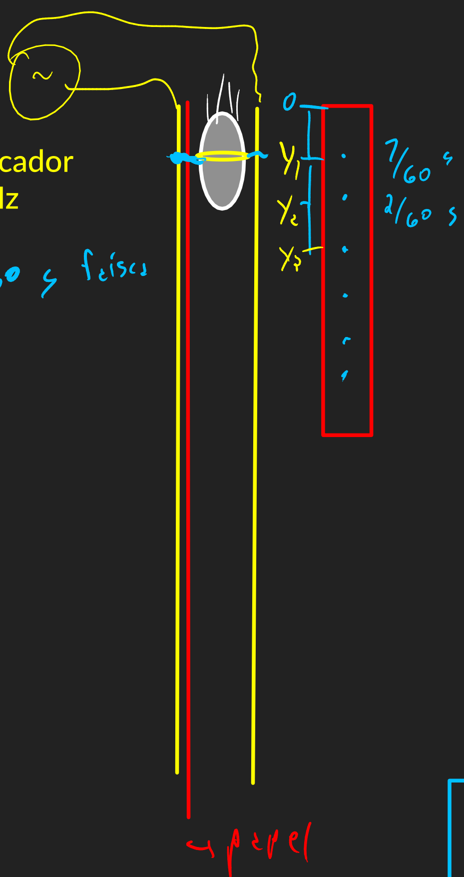
$$\begin{bmatrix} \langle \varphi_1, \varphi_1 \rangle & \langle \varphi_2, \varphi_1 \rangle & \cdots & \langle \varphi_n, \varphi_1 \rangle \\ \langle \varphi_1, \varphi_2 \rangle & \langle \varphi_2, \varphi_2 \rangle & \cdots & \langle \varphi_n, \varphi_2 \rangle \\ \vdots & & \ddots & \vdots \\ \langle \varphi_1, \varphi_n \rangle & \cdots & \cdots & \langle \varphi_n, \varphi_n \rangle \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix} = \begin{bmatrix} \langle y, \varphi_1 \rangle \\ \langle y, \varphi_2 \rangle \\ \vdots \\ \langle y, \varphi_n \rangle \end{bmatrix}$$

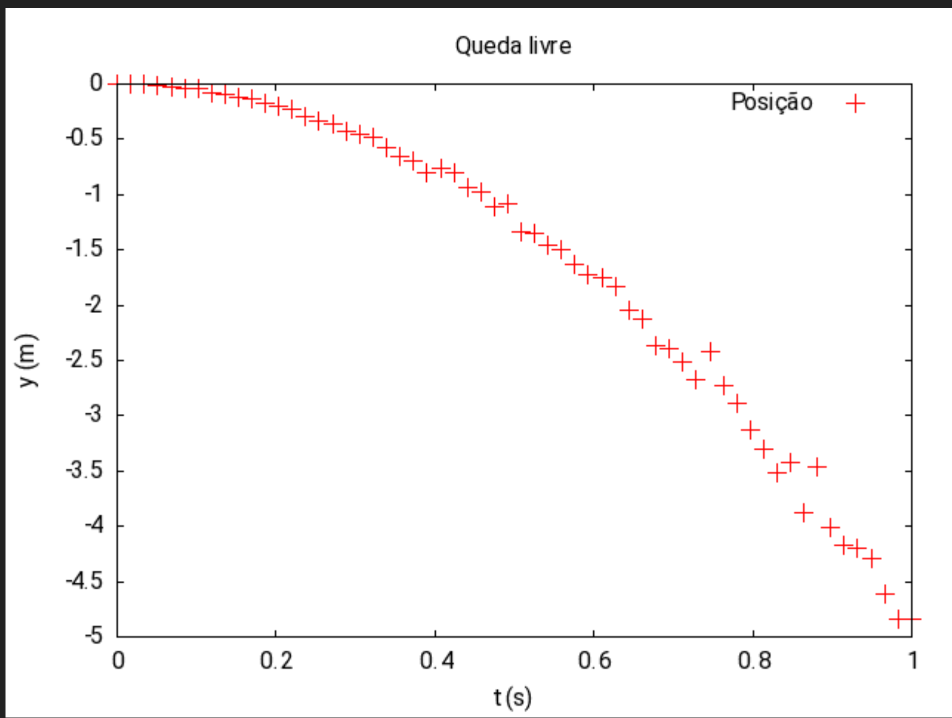
$$f(x) = \alpha_1 \varphi_1(x) + \alpha_2 \varphi_2(x) \ldots$$

Exemplo: Experimento da queda do ovo

Faiscador 60Hz

$1/60$ s faísca





$$\boxed{h(t) = y_0 + v_0 t + \frac{1}{2} a t^2}$$

MODELO

Parametros

$$\langle f, g \rangle = \sum_{i=1}^{60} f(t_i)\, g(t_i)$$

$$\langle t, t^2 \rangle = \left[\frac{1}{60}\left(\frac{1}{60}\right)^2\right] + \left[\frac{2}{60}\left(\frac{2}{60}\right)^2\right] + \cdots \left[\frac{60}{60}\left(\frac{60}{60}\right)^2\right]$$

$\{ g_1 = 1 \quad g_2 = t \quad g_3 = \frac{1}{2} t^2 \}$ ⇝ geradores do subespaço

$$h(t) = y_0 + v_0 t + a\left[\frac{1}{2} t^2\right]$$

$$h(t) = y_0 g_1 + v_0 g_2 + a g_3$$

$$\begin{cases} \langle y - h, g_1 \rangle = 0 \\ \langle y - h, g_2 \rangle = 0 \\ \langle y - h, g_3 \rangle = 0 \end{cases}$$

$$\begin{cases} y_0 \langle g_1, g_1 \rangle + v_0 \langle g_2, g_1 \rangle + a \langle g_3, g_1 \rangle = \langle y, g_1 \rangle \\ y_0 \langle g_1, g_2 \rangle + v_0 \langle g_2, g_2 \rangle + a \langle g_3, g_2 \rangle = \langle y, g_2 \rangle \\ y_0 \langle g_1, g_3 \rangle + v_0 \langle g_2, g_3 \rangle + a \langle g_3, g_3 \rangle = \langle y, g_3 \rangle \end{cases}$$

$$\underbrace{\begin{bmatrix} \langle g_1, g_1 \rangle & \langle g_2, g_1 \rangle & \langle g_3, g_1 \rangle \\ \langle g_1, g_2 \rangle & \langle g_2, g_2 \rangle & \langle g_3, g_2 \rangle \\ \langle g_1, g_3 \rangle & \langle g_2, g_3 \rangle & \langle g_3, g_3 \rangle \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} y_0 \\ v_0 \\ a \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} \langle y, g_1 \rangle \\ \langle y, g_2 \rangle \\ \langle y, g_3 \rangle \end{bmatrix}}_{b}$$

$$A x = b$$

Preciso calcular os seguintes produtos internos

$$\begin{cases} \langle g_1, g_1 \rangle = 60 \quad \langle g_2, g_1 \rangle = 30 \quad \langle g_3, g_1 \rangle = 210 \quad \langle y, g_1 \rangle \\ \langle g_2, g_2 \rangle \quad \langle g_3, g_2 \rangle \quad \langle y, g_2 \rangle \\ \langle g_3, g_3 \rangle \quad \langle y, g_3 \rangle \end{cases}$$

$x(t) = A \operatorname{sen}(\omega t) + B \cos(\omega t)$

(ω é dado)

$B = \{\operatorname{sen}(\omega t), \cos(\omega t)\}$

Considere o espaço vetorial $\mathbb{R}^3$ com o produto interno usual e a base $\mathrm{B} = \{\boldsymbol{v}_1 = (1,1,0), \boldsymbol{v}_2 = (0,1,5), \boldsymbol{v}_3 = (0,2,1)\}$.

1. Obtenha $\langle \boldsymbol{v}_1, \boldsymbol{v}_2 \rangle$, $\langle \boldsymbol{v}_1, \boldsymbol{v}_3 \rangle$
2. Obtenha uma base ortonormal aplicando o processo de Gram-Schmidt na base B
3. Escreva as coordenadas do vetor $\boldsymbol{u} = (1,2,2)$

$\langle v_1, v_2 \rangle = \langle (1,1,0), (0,1,5) \rangle = 1$

G = ortonormal

$G = \{g_1, g_2, g_3\}$

$u = (1,2,2) = \alpha g_1 + \beta g_2 + \gamma g_3$

$\alpha = \langle u, g_1 \rangle$

$\beta = \langle u, g_2 \rangle$

$\gamma = \dfrac{\langle u, g_3 \rangle}{\langle g_3, g_3 \rangle}$

$\langle u, g_1 \rangle = \langle \alpha g_1 + \beta g_2 + \gamma g_3, g_1 \rangle$

$$= \alpha \underbrace{\langle g_1, g_1 \rangle}_{1} + \beta \overbrace{\langle g_2, g_1 \rangle}^{0} + \gamma \underbrace{\langle g_3, g_1 \rangle}_{0}$$

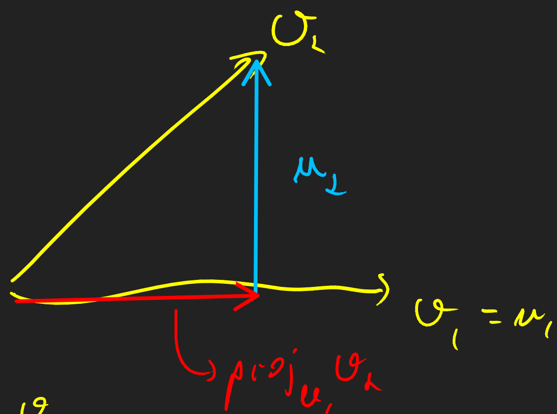
$\|g_1\| = 1$ $\qquad g_2 \perp g_1 \qquad g_3 \perp g_1$

$$B = \{ v_1, v_2, v_3 \}$$

$u_1 = v_1$

$u_2 = v_2 - \text{proj}_{u_1} v_2$

$u_3 = v_3 - \text{proj}_{u_2} v_3 - \text{proj}_{u_1} v_3$



$$\text{proj}_u v = \frac{\langle u, v \rangle}{\langle u, u \rangle} u$$

$B_1 = \{ u_1, u_2, u_3 \}$; $G = \left\{ \frac{u_k}{\|u_k\|}, \dots \right\}$

ortogonal $\qquad$ ortonormal