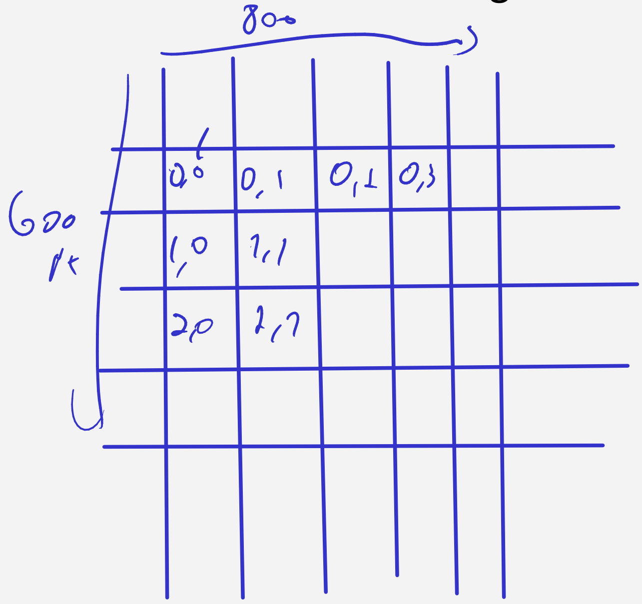
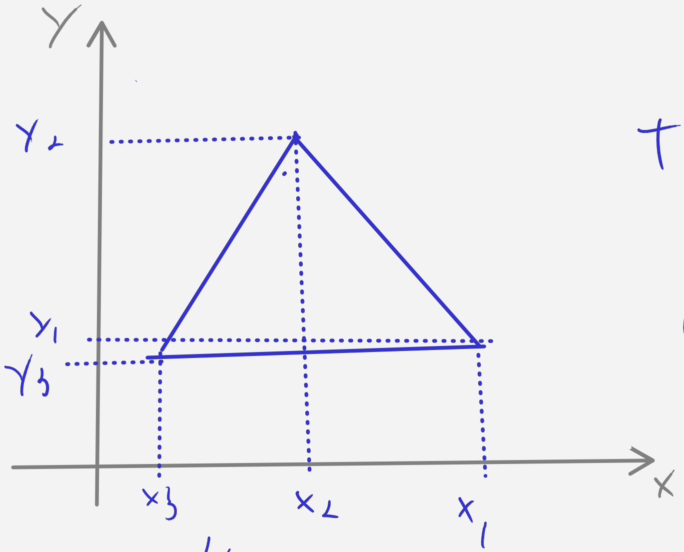
Notas de aula ECT2202 T04 2021-03-30 Aula 18 — Transf. Lineares e geometria

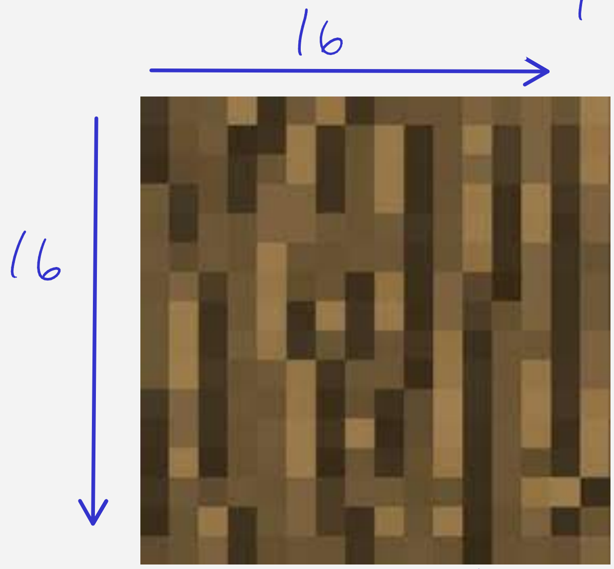
(300, 420) (300, 580)





Triângulo$((x_1, y_1), (x_2, y_2), (x_3, y_3))$

vértices

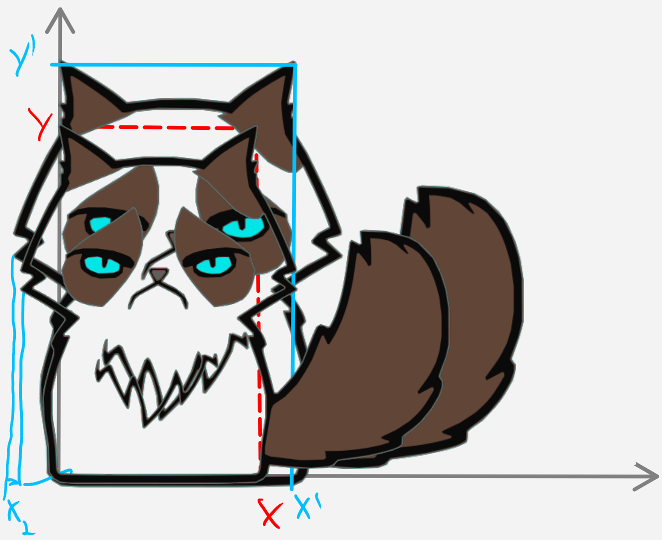


Textura

Transf.

Transformações na geometria do $\mathbb{R}^2$

Escala (ampliar/reduzir)



$$(x', y') = S(x, y) = (\alpha x, \alpha y)$$

$\alpha > 1$ aumento

$0 < \alpha < 1$ redução.

Matriz de escala: (simétrica)

$$[S] = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} \Rightarrow \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \alpha x \\ \alpha y \end{bmatrix}$$

Assimétrica:

$$[S] = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix}$$

$\alpha_1 = 1$

$\alpha_2 \cong 1/2$

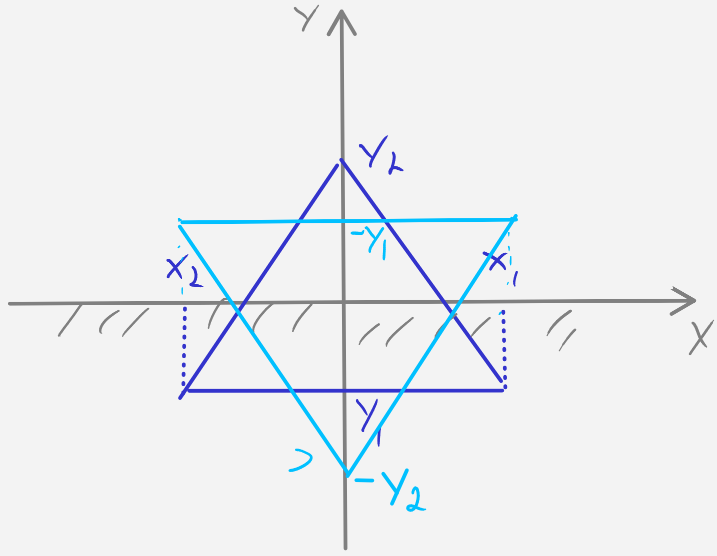
Em dimensão 3: $\mathbb{R}^3$

$$(x', y', z') = S(x, y, z) = (\alpha_1 x, \alpha_2 y, \alpha_3 z)$$

$$[S] = \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix}$$

Reflexões no $\mathbb{R}^2$

Reflexão no eixo x



Triangulo:

(x_1, y_1)

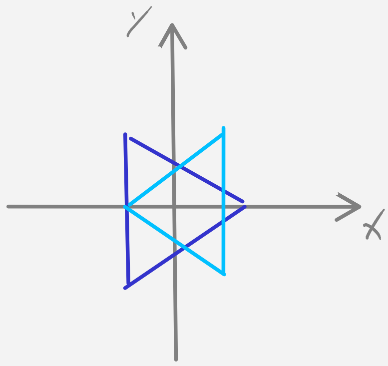
$(0, y_2)$

(x_2, y_1)

$$(x', y') = E_x(x, -y)$$

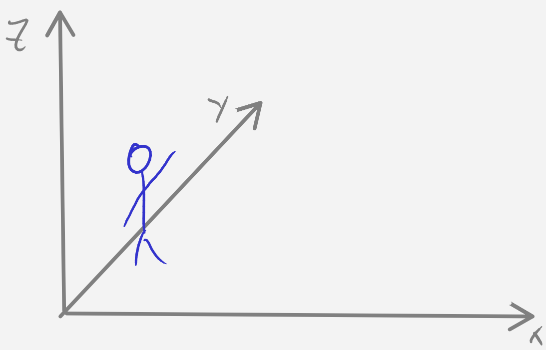
$$[E_x] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$[E_y] = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



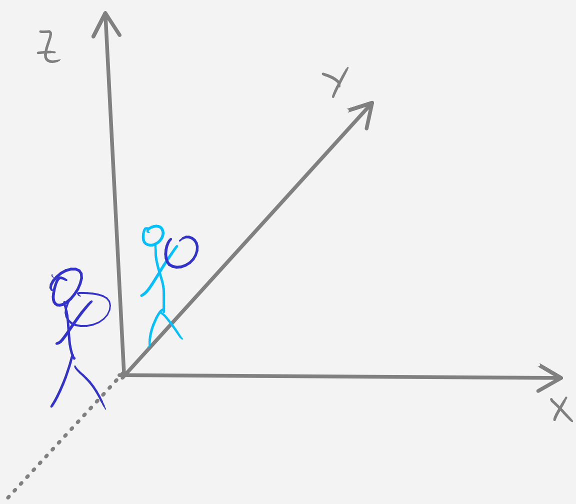
$\mathbb{R}^3$:

Reflexão em torno do plano xy



$$(x', y', z') = E_{xy}(x, y, z)$$
$$= (x, y, -z)$$

$$[E_{xy}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

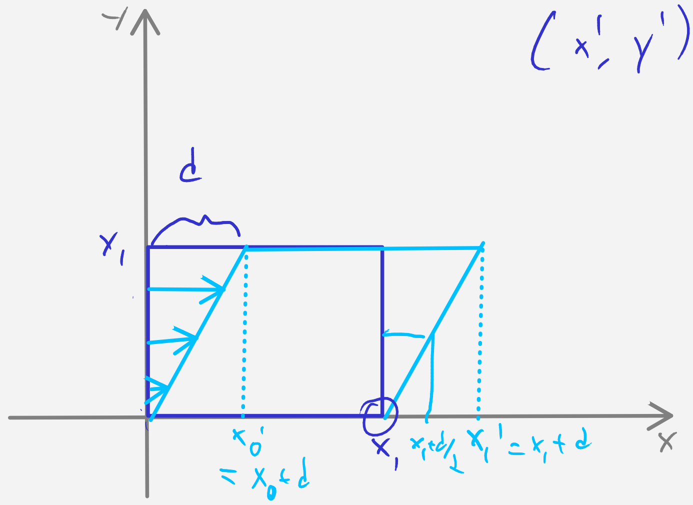


Reflexão em xz

$$(x', y', z') = E_{xz}(x, y, z)$$
$$= (x, -y, z)$$

$$[E_{xz}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Cisalhamento no $\mathbb{R}^2$
(Shear)

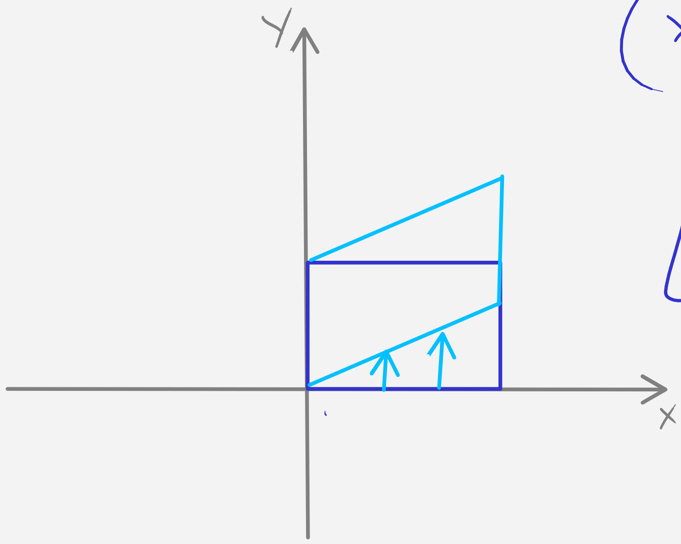


$$(x', y') = C_y(x, y)$$
$$= (x + \alpha y, y)$$
$$= \left(x + \frac{d}{y_1} y, y\right)$$

$$[C_y] = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + \alpha y \\ y \end{bmatrix}$$

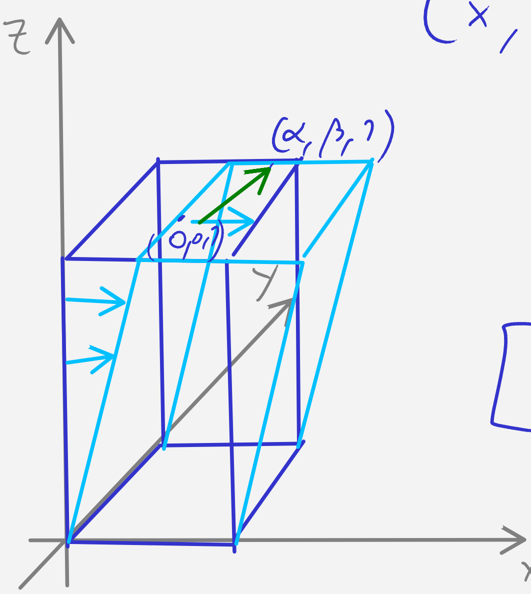
Cisalhamento no eixo x (direção y)



$$(x', y') = C_x(x, y+\alpha x)$$

$$[C_x] = \begin{bmatrix} 1 & 0 \\ \alpha & 1 \end{bmatrix}$$

Cisalhamento no $\mathbb{R}^2$



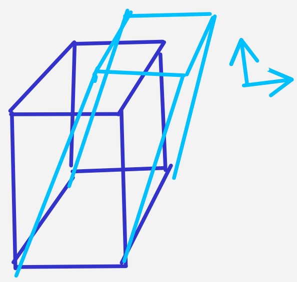
$$(x', y', z') = C_{z,x}(x, y, z)$$

$$= (x+\alpha z, y, z)$$

$$[C_{z,x}] = \begin{bmatrix} 1 & 0 & \alpha \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

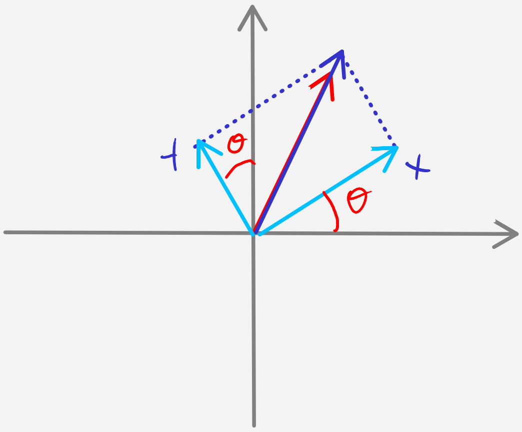
$$C_z(x, y, z) = (x+\alpha z, y+\beta z, z)$$

$$[C_z] = \begin{bmatrix} 1 & 0 & \alpha \\ 0 & 1 & \beta \\ 0 & 0 & 1 \end{bmatrix}$$



$$C_{z,x}(0,0,1) = (0+\alpha 1, 0, 1)$$

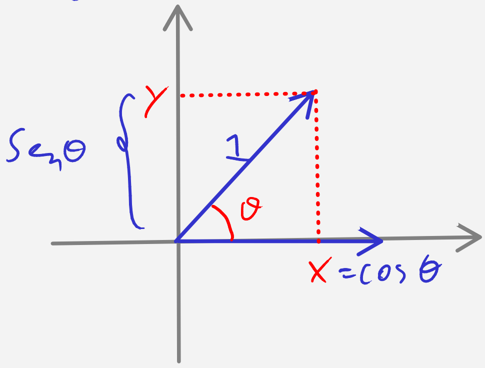
Rotação no $\mathbb{R}^2$



Observar a ação da rotação nos vetores da base:

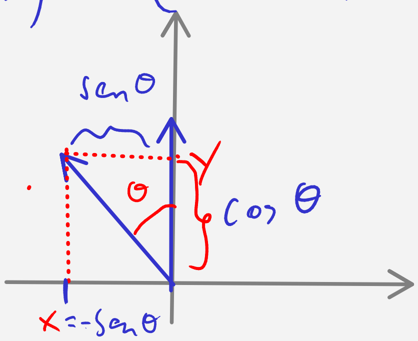
$$R_\theta(x,y) = x\,R_\theta(1,0) + y\,R_\theta(0,1)$$

$$R_\theta(1,0) = (\cos\theta, \operatorname{sen}\theta)$$



$$\begin{bmatrix} \cos\theta \\ \operatorname{sen}\theta \end{bmatrix}$$

$$R_\theta(0,1) = (-\operatorname{sen}\theta, \cos\theta)$$



$$\begin{bmatrix} -\operatorname{sen}\theta \\ \cos\theta \end{bmatrix}$$

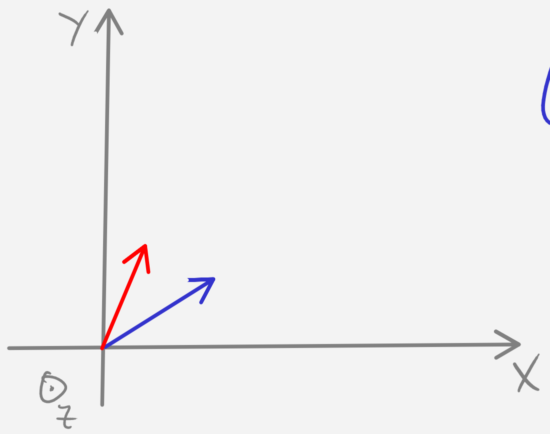
$$[R_\theta] = \begin{bmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{bmatrix}$$

$$\begin{aligned} R_\theta(x,y) &= x\,R_\theta(1,0) + y\,R_\theta(0,1) \\ &= x(\cos\theta, \operatorname{sen}\theta) + y(-\operatorname{sen}\theta, \cos\theta) \\ &= (x\cos\theta - y\operatorname{sen}\theta,\ x\operatorname{sen}\theta + y\cos\theta) \end{aligned}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotações em $\mathbb{R}^3$

Rotação em torno do eixo z

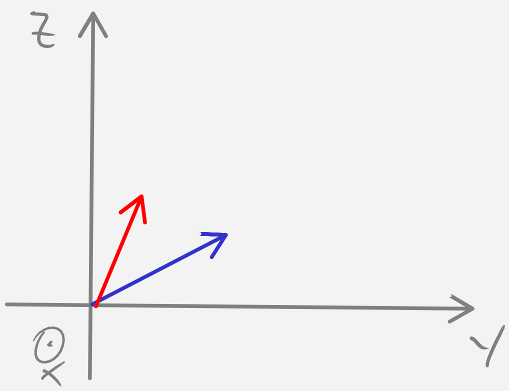


$$[R_{z,\theta}] = \begin{bmatrix} \cos\theta & -\text{sen}\,\theta & 0 \\ \text{sen}\,\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\text{sen}\,\theta & 0 \\ \text{sen}\,\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

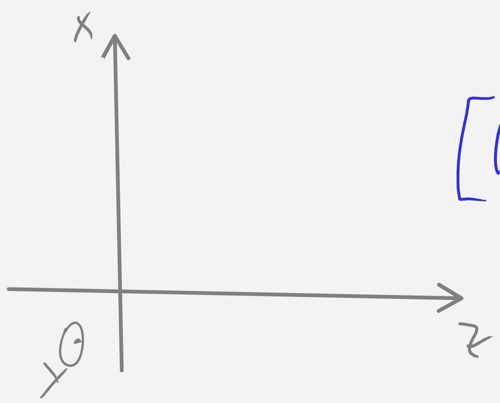
$$\hat{x} \times \hat{y} = \hat{z}$$
$$\hat{y} \times \hat{z} = \hat{x}$$
$$\hat{z} \times \hat{x} = \hat{y}$$

Rotação em torno do eixo x



$$[R_{x,\theta}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\text{sen}\,\theta \\ 0 & \text{sen}\,\theta & \cos\theta \end{bmatrix}$$

Rotação em torno do eixo y



$$[R_{y,\theta}] = \begin{bmatrix} \cos\theta & 0 & \text{sen}\,\theta \\ 0 & 1 & 0 \\ -\text{sen}\,\theta & 0 & \cos\theta \end{bmatrix}$$

$$\begin{array}{cccccc}
\cos\theta & -\operatorname{sen}\theta & 0 & \cos\theta & -\operatorname{sen}\theta & 0 \\
\operatorname{sen}\theta & \cos\theta & 0 & \operatorname{sen}\theta & \cos\theta & 0 \\
0 & 0 & 1 & 0 & 0 & 1 \\
\cos\theta & -\operatorname{sen}\theta & 0 & \cos\theta & -\operatorname{sen}\theta & 0 \\
\operatorname{sen}\theta & \cos\theta & 0 & \operatorname{sen}\theta & \cos\theta & 0 \\
0 & 0 & 1 & 0 & 0 & 1
\end{array}$$

$$[R_{y,\theta}] = \begin{bmatrix} \cos\theta & 0 & \operatorname{sen}\theta \\ 0 & 1 & 0 \\ -\operatorname{sen}\theta & 0 & \cos\theta \end{bmatrix}$$

$-\operatorname{sen}\theta$

$\operatorname{sen}\theta \quad \cos\theta$

Translação em $\mathbb{R}^3$

$$(x', y', z') = T(x, y, z)$$
$$= (x+\alpha, y+\beta, z+\gamma)$$

NÃO É TRANSFORMAÇÃO LINEAR.
(TRANSFORMAÇÃO AFIM)

COORDENADAS HOMOGÊNEAS

$$(x, y, z, 1)$$

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \alpha & \beta & \gamma & 1 \end{bmatrix}$$

Em coordenjadas homogêneas, a transalção é uma transformação linear

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \alpha & \beta & \gamma & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x+\alpha \\ y+\beta \\ z+\gamma \\ 1 \end{bmatrix}$$